SuccessClap

Best Coaching for UPSC Mathematics

UPSC CSE Mathematics: Previous Year Questions: Calculus

2025

1) A rectangular sheet of metal of length 6 meters and width 2 meters is given. Four equal squares are removed from the four corners. The sides of this sheet are now folded up to form an open rectangular box. Find approximately the height of the box, such that the volume of the box is maximum.

2) Given that $f(x+y)=f(x)f(y)$ for all real $x, y, f(x) \neq 0$ for any real x and $f'(0)=2$. Show that for all real $x, f'(x)=2f(x)$. Hence find $f(x)$.

3) Using Mean Value $\frac{\pi}{6}+\frac{\sqrt{3}}{15}<\sin^{-1}\left(\frac{3}{5}\right)<\frac{\pi}{6}+\frac{1}{8}$

4) Evaluate $\iint_R ydxdy$, where R is the region bounded by $y=x$ and $y=4x-x^2$

5) If $u(x,y)=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$, where f and g are arbitrary functions, then show that
I. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=xf\left(\frac{y}{x}\right)$,
II. $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$.

6) If $f(x,y)=\begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & \text{when } (x,y)\neq(0,0) \\ 0, & \text{when } (x,y)=(0,0)\end{cases}$ then find $f_{xy}(0,0)$ and $f_{yx}(0,0)$

2024

1) Discuss the continuity of the function $f(x)=\begin{cases}\frac{1}{1-e^{-1/x}}, & x\neq 0 \\ 0, & x=0\end{cases}$

for all values of x.

2) Expand $\ln(x)$ in powers of $(x-1)$ by Taylor's theorem and hence find the value of In (1.1) correct up to four decimal places.

3) Using double integration, find the area lying inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$.

4) Find the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α.

5) If $u=(x+y)/(1-xy)$ and $v=\tan^{-1}x+\tan^{-1}y$, then find $\partial(u,v)/\partial(x,y)$. Are u and v functionally related? If yes, find the relationship.

2023

1) Find the values of p and q for which $\lim_{x\to 0}\frac{x(1+p\cos x)-q\sin x}{x^3}$ exists and equals

2) Examine the convergence of the integral $\int_0^1 \frac{\log x}{1+x}dx$

3) Evaluate the triple integral which gives the volume of the solid enclosed between the two paraboloids $Z = 5(x^2+y^2)$ and $Z = 6-7x^2-y^2$.

4) Trace the curve $y^2(x^2-1) = 2x-1$

5) Justify whether (0,0) is an extreme point for the function $f(x,y) = 2x^4-3x^2y+y^2$

2022

1) Evaluate $\lim_{x\to\infty}(e^x+x)^{\frac{1}{x}}$

2) Examine the convergence of $\int_0^2 \frac{dx}{(2x-x^2)}$.

3) A wire of length l is cut into two parts which are bent in the form of a square and a circle respectively. Using Lagrange's method of undetermined multipliers, find the least value of the sum of the areas so formed.

4) Use double integration to calculate the area common to the circle $x^2+y^2 = 4$ and the parabola $y^2 = 3x$.

5) Trace the curve $y^2x^2 = x^2-a^2$, where a is a real constant.

2021

1) Given: $\Delta(x) = \begin{vmatrix} f(x+\alpha) & f(x+2\alpha) & f(x+3\alpha) \\ f(\alpha) & f(2\alpha) & f(3\alpha) \\ f'(\alpha) & f'(2\alpha) & f'(3\alpha) \end{vmatrix}$

where f is a real valued differentiable function and α is a constant. Find $\lim_{x\to 0}\frac{\Delta(x)}{x}$

2) Show that between any two roots of $e^x\cos x = 1$, there exists at least one root of $e^x\sin x - 1 = 0$

3) Given $f(x,y) = |x^2-y^2|$. Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$.Hence show that $f_{xy}(0,0) = f_{yx}(0,0)$.

4) If $u = x^2+y^2$, $v = x^2-y^2$, where $x = r\cos\theta, y = r\sin\theta$, then find $\frac{\partial(u,v)}{\partial(r,\theta)}$.

5) If $\int_0^x f(t)\,dt = x + \int_x^1 tf(t)\,dt$ then find the value of $f(1)$.

6) Express $\int_a^b (x-a)^m(b-x)^n\,dx$ in terms of Beta function.

7) Show that the entire area of the Asteroid: $x^{2/3}+y^{2/3} = a^{2/3}$ is $\frac{3}{8}\pi a^2$.

2020

1) Evaluate $\lim_{x\to\frac{\pi}{4}}(\tan x)^{\tan 2x}$

2) Find all the asymptotes of the curve $(2x+3)y=(x-1)^2$

3) Evaluate $\int_0^1 \tan^{-1}\left(1-\frac{1}{x}\right)dx$

4) Consider the function $f(x)=\int_0^x (t^2-5t+4)(t^2-5t+6)dt$

   (i) Find the critical points of the function $f(x)$
   (ii) Find the points at which local minimum occurs.
   (iii) Find the points at which local maximum occurs.
   (iv) Find the number of zeros of the function $f(x)$ in [0,5]

5) Find an extreme value of the function $u=x^2+y^2+z^2$ subject to the condition $2x+3y+5z=30$ by using Lagrange's method of undetermined multiplier.

2019

1) Let $f:\left[0,\frac{\pi}{2}\right]\to R$ be a continuous function such that $f(x)=\frac{\cos^2 x}{4x^2-\pi^2}$, $0\le x\le\frac{\pi}{2}$. Find the value of $f\left(\frac{\pi}{2}\right)$

2) Let $f:D(\subseteq R^2)\to R$ be a function and $(a,b)\in D$. If $f(x,y)$ is continuous at (a,b), then show the functions $f(x,b)$ and $f(a,y)$ are continuous at $x=a$ and at $y=b$ respectively.

3) Is $f(x)=|\cos\ x|+|\sin\ x|$ differentiable at $x=\frac{\pi}{2}$? If yes, then find its derivative at $x=\frac{\pi}{2}$ $x=\frac{\pi}{2}$ If no, then a proof of it.

4) Find the maximum and the minimum value of the function $f(x)=2x^3-9x^2+12x+6$ on the interval [2,3]

5) If $u=\sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $\sin^2\ u$ is a homogeneous function of x and y of degree $-\frac{1}{6}$ hence show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan\ u}{12}\left(\frac{13}{12}+\frac{\tan^2\ u}{12}\right)$

6) Use Jacobian method, show that if $f'(x)=\frac{1}{1+x^2}$ and $f(0)=0$ then $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$

2018

1) Determine if $\lim_{z\to1}(1-z)\tan\frac{\pi z}{2}$ exists or not. If limit exists, then find its value.

2) Find the limit $\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=0}^{n-1}\sqrt{n^2-r^2}$.

3) Find the shortest distance from the point $(1,0)$ to the parabola $y^2=4x$

4) The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolves about the x-axis. Find the volume of the solid of revolution.

5) Let $f(x,y)=\begin{cases}xy^2, & y>0\\ -xy^2, & y\le 0\end{cases}$. Determine which of $\frac{\partial f}{\partial x}(0,1)$ and $\frac{\partial f}{\partial y}(0,1)$ exists and which does not exist.

6) Find the maximum and the minimum values of x^4-5x^2+4 on the interval [2,3].

7) Evaluate the integral $\int_0^a\int_{x/a}^x \frac{x\,dydx}{x^2+y^2}$

2017

1) Integrate the function $f(x,y) = xy(x^2+y^2)$ over the domain $R: \{-3 \le x^2 - y^2 \le 3, 1 \le xy \le 4\}$

2) Find the volume of the solid above the xy-plane and directly below the portion of the elliptic paraboloid $x^2 + \frac{y^2}{4} = z$ which is cut off by the plane $z = 9$

3) If $f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$ calculate $\frac{\partial^2 f}{\partial x\,\partial y}$ and $\frac{\partial^2 f}{\partial y\,\partial x}$ at $(0,0)$

4) Examine if the improper integral $\int_0^3 \frac{2x\,dx}{(1-x^2)^{2/3}}$, exists.

5) Prove that $\frac{\pi}{3} \le \iint_D \frac{dxdy}{\sqrt{x^2+(y-2)^2}} \le \pi$ where D is the unit disc.

2016

1) Evaluate: $I = \int_0^1 \sqrt[3]{x\log\left(\frac{1}{x}\right)}\,dx$

2) Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $x + y - z = 0$

3) Let $f(x,y) = \begin{cases} \frac{2x^4y - 5x^2y^2 + y^5}{(x^2+y^2)^2}, & x,y) \neq (0,0) \\ 0 & ,x,y) = (0,0) \end{cases}$

Find a $\delta > 0$ such that $|f(x,y) - f(0,0)| < 0.01$, whenever $\sqrt{x^2+y^2} < \delta$

4) Find the surface area of the plane $x + 2y + 2z = 12$ cut off by $x = 0, y = 0$ and $x^2 + y^2 = 16$

5) Evaluate $\iint_R f(x,y)dxdy$, over the rectangle $R = [0,1;0,1]$ where

$$f(x,y) = \begin{cases} x+y, & \text{if } x^2 < y < 2x^2 \\ 0, & \text{elsewhere} \end{cases}$$

2015

1) Evaluate the following limit $\lim_{x\to a}\left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$

2) Evaluate the following integral: $\int_{\pi/6}^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}}\,dx$

3) A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base.

4) Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point $(2,1,3)$

5) Evaluate the integral $\iint_R (x-y)^2\cos^2(x+y)dxdy$ where R is the rhombus with successive vertices as $(\pi,0), (2\pi,\pi), (\pi,2\pi), (0,\pi)$

6) Evaluate $\iint_R \sqrt{|y - x^2|}\;dxdy$ where $R = [-1,1;\ 0,2]$

7) For the function $f(x,y) = \begin{cases} \frac{x^2 - x\sqrt{y}}{x^2+y}, & (x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$

Examine the continuity and differentiability.

2014

1) Prove that between two real roots $e^x \cos x + 1 = 0$, a real root of $e^x \sin x + 1 = 0$ lies.

2) Evaluate: $\int_0^1 \frac{\log_e(1+x)}{1+x^2} dx$

3) By using the transformation $x + y = u,\ y = uv$ evaluate the integral $\iint \{xy(1-x-y)\}^{\frac{1}{2}} dxdy$ taken over the area enclosed by straight lines $x = 0, y = 0$ and $x + y = 1$.

4) Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a.

5) Find the maximum or minimum values of $x^2 + y^2 + z^2$ subject to the condition $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$ interpret result geometrically.

2013

1) Evaluate $\int_0^1 \left(2x\sin\frac{1}{x} - \cos\frac{1}{x} dx\right)$

2) Using Lagrange's multiplier find the shortest distance between the line $y = 10 - 2x$ and ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

3) Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function $f(x,y) = \begin{cases} \frac{xy^3}{x+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$

Also discuss the continuity of f_{xy} and f_{yx} at $(0,0)$.

4) Evaluate $\iint_D xy dA$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

2012

1) Define a function f of two real variables in the plane by $f(x,y) = \begin{cases} \frac{x^3\cos\frac{1}{y} + y^3\cos\frac{1}{x}}{x^2+y^2} \text{ for } x, y \neq 0 \\ 0, \text{ otherwise} \end{cases}$

Check the continuity and differentiability of f at $(0,0)$.

2) Let p and q be positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$ show that for real numbers $a, b \geq 0 \quad ab \leq \frac{a^p}{p} + \frac{b^q}{q}$

3) Find the point of local extreme and saddle points of the function f for two variables defined by $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$

4) Define a sequence S_n of real numbers by $S_n = \sum_{i=1}^{n} \frac{(\log(n+i)-\log n)^2}{n+1}$
Does $\lim_{n\to\infty} S_n$ exist? If so compute the value of this limit and justify your answer

5) Compute the volume of the solid enclosed between the surfaces
$x^2 + y^2 = 9$ and $x^2 + z^2 = 9$.

6) Find all the real values of p and q so that the integral $\int_0^1 x^p \left(\log \frac{1}{x}\right)^q dx$ converges

2011

1) Find $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^3+y^3}$ if it exists

2) Let f be a function defined on $\mathbb{R}$ such that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x in $\mathbb{R}$
How large can $f(2)$ possibly be?

3) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3,1,-1)$

4) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ above the xy-plane and inside the cylinder $x^2 + y^2 = 2x$.

5) Evaluate: (i) $\lim_{x\to 2} f(x)$ Where $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ \pi, & x = 2 \end{cases}$

(ii) $\int_0^1 \ln\, x dx.$

2010

1) A twice differentiable function $f(x)$ is such that $f(a) = 0 = f(b)$ and $f(c) > 0$ for $a < c < b$ prove that there be is at least one point ξ , $a < \xi < b$ for which $f''(\xi) < 0$

2) Dose the integral $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}$ exist if so, find its value

3) Show that a box (rectangular parallelepiped) of maximum volume V with prescribed surface area is a cube.

4) Let D be the region determine by the inequalities $x > 0, y > 0, z < 8$ and $z > x^2 + y^2$.
Compute $\iiint_D 2x dx dy dz$

5) If $f(x, y)$ is a homogeneous function of degree n in x and y, and has continuous first and second order partial derivatives then show that
(i) $x \frac{\partial^2 f}{\partial x} + y \frac{\partial^2 f}{\partial y} = nf$
(ii) $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial \times \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1)f$

6) Show that the function $f(x) = [x^2] + |x - 1|$ is Riemann integrable in the interval $[0,2]$, where $[\alpha]$ denotes the greatest integer less than or equal to α. Can you give an example of a function that is not Riemann integrable on $[0,2]$? Compute $\int_0^2 f(x)dx$, where $f(x)$ is as above.

2009

1) Suppose the f'' is continuous on $[1,2]$ and that f has three zeroes in the interval $(1,2)$. Show that f'' has at least one zero in the interval $(1,2)$.

2) If f is the derivative of some function defined on $[a,b]$, prove that there exists a number $\eta \in [a,b]$ such that $\int_a^b f(t)dt = f(\eta)(b-a)$

3) If $x = 3 \pm 0.01$ and $y = 4 \pm 0.01$ with approximately, what accuracy can you calculate the polar coordinate r and θ of the point $P(x,y)$. Express your estimates as percentage changes of the values that r and θ have at the point $(3,4)$

4) A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth atmosphere and its surface beings to heat. After one hour, the temperature at the point (x,y,z) on the probe surface is given by $T(x,y,z) = 8x^2 + 4yz - 16z + 1600$
Find the hottest point on the probe surface.

5) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & \text{if} \quad (x,y) \neq (0,0) \\ 0, & \text{if} \quad (x,y) = (0,0) \end{cases}$$

Is f continuous at $(0,0)$? Compute partial derivatives of f at any point (x,y), if exist.

6) Evaluate $I = \iint_S xdydz + dzdx + xz^2dxdy$ where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

2008

1) Find the value of $\lim_{x\to 1} \ln(1-x)\cot\frac{\pi x}{2}$.

2) Evaluate $\int_0^1 (x \ln x)^3 dx$

3) Determine the maximum and minimum distances of the origin from the curve given by the equation $3x^2 + 4xy + 6y^2 = 140$

4) Evaluate the double integral $\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$ by changing the order of integration

5) Obtain the volume bounded by the elliptic paraboloid given by the equations
$z = x^2 + 9y^2$ & $z = 18 - x^2 - 9y^2$

2007

1) Let $f(x)$, $(x \in (-\pi,\pi))$ be defined by $f(x) = \sin|x|$. Is f continuous on $(-\pi,\pi)$? If it is continuous, then is it differentiable on $(-\pi,\pi)$?

2) A figure bounded by one arch of a cycloid $x = a(t - \sin t), y = a(1 - \cos t), t \in [0,2\pi]$ and the x-axis is revolved about the x-axis. Find the volume of the solid of revolution

3) Find a rectangular parallelepiped of greatest volume for a give total surface area S using Lagrange's method of multipliers

4) Prove that if $z = \phi(y+ax) + \psi(y-ax)$ then $a^2\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2} = 0$ for any twice differentiable φ and ψ is a constant.

5) Show that $e^{-x}x^n$ is bounded on $[0,\infty)$ for all positive integral values of n. Using this result show that $\int_0^\infty e^{-x}x^n dx$ exists.

2006

1) Find a and b so that $f'(2)$ exists where $f(x) = \begin{cases} \frac{1}{|x|}, & \text{if } |x| > 2 \\ a + bx^2 & \text{if } |x| \leq 2 \end{cases}$

2) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma function and hence evaluate the integral $\int_0^1 x^6\sqrt{(1-x^2)}dx$

3) Find the values of a and b such that $\lim_{x\to 0} \frac{a\sin^2 x + b\log\cos x}{x^4} = \frac{1}{2}$.

4) If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ show that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\,\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$

5) Change the order of integration in $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dydx$ and hence evaluate it.

6) Find the volume of the uniform ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

2005

1) Show that the function given below is not continuous at the origin
$f(x,y) = \begin{cases} 0 \text{ if } xy = 0 \\ 1 \text{ if } xy \neq 0 \end{cases}$

2) Let $R^2 \to R$ be defined as $f(x,y) = \frac{xy}{\sqrt{(x^2+y^2)}}, (x,y) \neq (0,0), f(0,0) = 0$ prove that f_x and f_y exist at $(0,0)$ but f is not differentiable at $(0,0)$.

3) If $u = x + y + z$, $uv = y + z$ and $uvw = z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

4) Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$ in terms of Beta function.

5) Evaluate $\iiint_V zdV$ where V is the volume bounded below by the cone $x^2 + y^2 = z^2$ and above by the sphere $x^2 + y^2 + z^2 = 1$ lying on the positive side of the y-axis.

6) Find the x-coordinate of the center of gravity of the solid lying inside the cylinder $x^2 + y^2 = 2ax$ between the plane $z = 0$ and the paraboloid $x^2 + y^2 = az$.

2004

1) Prove that the function f defined on $[0,4]$ $by\ f(x) = [x]$, greatest integer $\leq x, x \in [0,4]$ is integrable on $[0,4]$ and that $\int_0^4 f(x)dx = 6$

2) Shaw that $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$, $x > 0$.

3) Let the roots of the equation in $\lambda\ in\ (\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ be u, v, w proving that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = -2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$.

4) Prove that an equation of the form $x^n = \alpha$, where ne / N and $\alpha > 0$ is a real number, has a positive root.

5) Prove that $\int \frac{x^2+y^2}{p} dx = \frac{\pi ab}{4}[4 + (a^2 + b^2)(a^{-2} + b^{-2})]$ when the integral is taken round the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and p is length of three perpendicular from the center to the tangent.

6) If the function f is defined by $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, (x,y) \neq (0,0) \\ 0 , (x,y) = (0,0) \end{cases}$ then show that f possesses both the partial derivative at $(0,0)$ but it is not continuous thereat.

2003

1) Let f be a real function defined as follow:
$f(x) = x, \quad -1 \leq x < 1$
$f(x+2) = x, \quad \forall x \in R$
Show that f is discontinuous at every odd integer.

2) For all real numbers $x, f(x)$ is given as
$f(x) = \begin{cases} e^x + a \sin x, & x < 0 \\ b(x-1)^2 + x - 2, & x \geq 0 \end{cases}$.
Find values of a and b for which f is differentiable at $x = 0$.

3) A rectangular box , open at the top is to have a volume of 4 m^3. Using Lagrange's method of multipliers, find the dimension of the box so that the material of a given type required to construct it may be least.

4) Test the convergent of the integrals
(i) $\int_0^1 \frac{dx}{x^{\frac{1}{3}}(1+x^2)}$ (ii) $\int_0^\infty \frac{\sin^2 x}{x^2} dx$

5) Evaluate the integral $\int_0^a \int_{\frac{y^2}{a}}^y \frac{ydxdy}{(a-x)\sqrt{ax-y^2}}$

6) Find the volume generated by revolving by the real bounded by the curves $(x^2 + 4a^2)y = 8a^3$, $2y = x$ and $x = 0$ about the y-axis.

2002

1) Show that $\frac{b-a}{\sqrt{1-a^2}} \leq \sin^{-1} b - \sin^{-1} a \leq \frac{b-a}{\sqrt{1-b^2}}$ for $0 < a < b < 1$

2) Show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy = \frac{\pi}{4}$

3) Let $f(x) = \begin{cases} x^p \sin 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$.
Obtain condition on p such that (i) f is continuous at $x = 0$ and
(ii) f is differentiable at $x = 0$

4) Consider the set of triangles having a given base and a given vertex angle show that the triangle having the maximum area will be isosceles

5) If the roots of the equation $(\lambda - u)^3 + (\lambda - v)^3 + (\lambda - w)^3 = 0$ in λ are x, y, z. show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = -\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$.

6) Find the center of gravity of the region bounded by the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$ and both axes in the first quadrant, the density being $\rho = kxy$ where k is constant.

2001

1) Let be defined on by setting $f(x)=x$ if x is rational and $f(x)=1-x$, if x is irrational show that is continuous at $x=\frac{1}{2}$ but is discontinuous at every other point.

2) Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}}dx$.

3) Find the equation of the cubic curve which has the same asymptotes as $2x(y-3)^2=3y(x-1)^2$ and which touches the x-axis at the origin and passes though the point $(1,1)$.

4) Find the maximum and minimum radii vectors of the section of the surface $(x^2+y^2+z^2)^2 = a^2x^2+b^2y^2+c^2z^2$ by the plane $lx+my+nz=0$

5) Evaluate $\iiint (x+y+z+1)^2 dxdydz$ over the region defined by $x\geq 0, y\geq 0, z\geq 0, x+y+z\leq 1$

6) Find the volume of the solid generated by revolving the cardioid $r=a(1-\cos\,\theta)$ about the initial line