SuccessClap
Best Coaching for UPSC Mathematics

Note: Solutions of all the questions are available in sequence in Crash Course. There is very less time for Mains exam and it is not possible to solve all problems, hence it is advisable to go through the solutions (Quick glance) of all these important questions at once. All topics, Important questions and important PYQ questions are covered.

Important Questions for UPSC Maths 2026

MECHANICS

Moment of Inertia

1) Calculate the moment of inertia of a uniform solid cylinder of mass M, radius R and length L with respect to a set of axes passing through the centre of the cylinder, where z-axis is the axis of the cylinder and ρ is the constant density at any point of the cylinder. Also find $\frac{L}{R}$ for which the moment of inertia about x - or y-axis will be minimum for a given mass of the cylinder.

2) Find the moment of inertia of a quadrant of an elliptic disk $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, of mass M about the line passing through its centre and perpendicular to its plane. Given that the density at any point is proportional to xy

3) Find the moment of inertia of a right circular solid cone about one of its slant sides (generator) in terms of its mass M, height h and the radius of base as a. (2022)

4) Prove that the moment of inertia of a triangular lamina ABC about any axis through in A its plane is $\frac{M}{6}(\beta^2+\beta\gamma+\gamma^2)$ where M is the mass of the lamina and β,γ are respectively the length of perpendiculars from B and C on the axis. (2020)

5) Show that the moment if inertia of an elliptic area of mass M and semi-axis a and b about a semi-diameter of length r is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. Further, prove that the moment of inertia about a tangent is $\frac{5M}{4}p^2$ where p is the perpendicular distance from the centre of the ellipse to the tangent. (2017)

6) Calculate the moment of inertia of a solid uniform hemisphere $x^2+y^2+z^2=a^2, z\geq 0$ with mass m about the OZ-axis. (2015)

7) For solid sphere A, B, C and D, each of mass m and radius a, are placed with their centers on the four corners of a square of side b. Calculate the moment of inertia of the system about a diagonal of the square. (2013)

8) A pendulum consists of a rod of length $2a$ and mass m; to one end of which a spherical bob of radius $\frac{a}{3}$ and mass $15\ \mathrm{m}$ is attached. Find the moment of inertia of the pendulum:

(a)About an axis through the other end of the rod and at right angle to the rod.

(b)About a parallel axis through the center of mass of the pendulum. [Given: the center of mass of the pendulum is $\frac{a}{12}$ above the centre of the sphere]. (2012)

9) Let a be the radius of the base of a right circular cone of height h and mass M. Find the moment of inertia of that right circular cone about a line through the vertex perpendicular to the axis. (2011)

10) A uniform lamina is bounded by a parabolic arc of latus rectum $4a$ and a double ordinate at a distance b from the vertex. If $b = \frac{a}{3}(7 + 4\sqrt{7})$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there. (2010)

11) The flat surface of a hemisphere of radius r is cemented to one flat surface of a cylinder of the same radius and of the same material. If the length of the cylinder be l and the total mass be m, show that the moment of inertia of the combination about the axis of the cylinder is given by $mr^2 \frac{\left(\frac{l}{2}+\frac{4}{15}r\right)}{\left(l+\frac{2r}{3}\right)}$. (2009)

12) A solid body of density ρ is in the shape of the solid formed by the revolution of the Cardioid $r = a(1 + \cos\ \theta)$ about the initial line Show that the moment of inertia about the straight line through the pole and perpendicular to the initial line is $\frac{352}{105}\pi\rho a^2$. (2003)

13) Find the moment of inertia of a circular wire about (i) a diameter and (ii) a line through the centre and perpendicular to its plane. (2002)

14) Determine the moment of inertia of a uniform hemisphere about its axis of symmetry and about an axis perpendicular to the axis of symmetry and though centre of the base. (2001)

15) Find the moment of inertia of an elliptic area about a line CP inclined at θ to the major axis and about a tangent parallel to CP where C is the centre of the ellipse. (2001)

16) Show that the moment of inertia of the area bounded by $r^2 = a^2\cos\ 2\theta$. (i) about its axis is $\frac{Ma^2}{16}\left(\pi - \frac{8}{3}\right)$. (ii) about a line through the origin, in its plane and perp. to its axis is $\frac{Ma^2}{16}\left(\pi + \frac{8}{3}\right)$. (iii) about a line through the origin and perp. to its plane is $\frac{Ma^2\pi}{8}$.

17) Find the moment of inertia of the triangle ABC about a perp, to its plane through A.

18) Show that the moment of inertia of a right solid cone whose height is h and radius of whose base is a, is $\frac{3Ma^2}{20}\left\{\frac{6h^2+a^2}{h^2+a^2}\right\}$ about a slant side and (3M/80) (h^2 + 4a^2) about a line through the center of gravity of the cone perpendicular to its axis.

D Alembert Principle

1) A rough uniform board of mass m and length 2a rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance covered by the board during this time.

2) A particle is constrained to move along a circle lying in the vertical xy-plane. With the help of the D'Alembert's principle, show that its equation of motion is $\ddot{x}y - \ddot{y}x - gx = 0$, where g is the acceleration due to gravity. (2021)

3) A uniform rod OA of length $2a$ free to turn about it end O revolves with angular velocity about the vertical OZ through O and is inclined at a constant angle α to OZ; find the value of α (2019)

4) A circular board is placed on a smooth horizontal plane and a boy runs round the edge of it at a uniform rate. What is the motion of the centre of the board? Explain. What happens if the mass of the board and boy are equal? (2008)

5) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time. $\sqrt{\frac{2M'a}{(M+M')g\sin\,\alpha}}$ Where a is the length of the plank? (2005) (2001)

6) A thin circular disc of mass M and radius a can turn freely about a thin axis OA which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity w about its end A. Show that the inclination ϕ to the vertical of the radius of the disc through O is $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$ unless $w^2 < \frac{g}{a}$ and then θ is zero. (2002) (1996)

7) What is D' Alembert's principle? An inextensible string of negligible mass hanging over a smooth page at A connects the mass m_1 on a frictionless inclined of angle θ to another mass m_2. Use D' Alembert's principle to prove that the mass will be in equilibrium if $m_2 = m_1 \sin\,\theta$. (1994)

8) A rod, of length $2a$, is suspended by a string of length l, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity, and their inclination to the vertical be θ and ϕ respectively, show that $\frac{3l}{a} = \frac{(4\tan\,\theta - 3\tan\,\phi)\sin\,\phi}{(\tan\,\phi - \tan\,\theta)\sin\,\theta}$.

9) A thin heavy disc can tum freely about an axis in its own plane, and this axis revolves horizontally with a uniform angular velocity ω about a fixed point on itself. Show that the inclination θ of the plane of the disc io the vertical is given by $\cos\ \theta = (gh/k^2\omega^2)$ where h is the distance of the centre of inentia of the disc from the axis and k is the radius of the gyration of the disc about the axis. If $\omega^2 < gh/k^2$, prove that the plane of the disc is vertical.

10) Two uniform spheres, each of mass M and radius a, are fimly fixed to the ends of two uniform thin rods, each of mass m and length l, and the other ends of the rods are freely hinged to a point O. The whole system revolves as in the Govemor of a steam-Engine, about a vertical line through O with the angular velocity ω. Show that when the motion is steady, the rods are inclined to the vertical at an angle θ given by the equation $\cos\ \theta = \frac{g}{\omega^2} \cdot \frac{M(l+a)+\frac{1}{2}ml}{M(l+a)^2+\frac{1}{3}ml^2}$.

11) A cannon of mass M, resting on a rough horizontal plane of coefficient of friction μ, is fired with such a charge that the relative velocity of the ball and cannon at the moment when it leaves the cannon is u. Show that the cannon will recoil a distance $\left(\frac{mu}{M+m}\right)^2 \frac{1}{2\mu g}$ along the plane, m being the mass of the ball.

Lagrangian Problems

1) A bead of mass m slides on a frictionless wire in the shape of a cycloid given by $x = a(\theta - \sin\ \theta), y = a(1 + \cos\ \theta), (0 \leq \theta \leq 2\pi)$. Find the Lagrangian function. Hence show that the equation of motion can be written as $\frac{d^2u}{dt^2} + \frac{g}{4a}u = 0$ where $u = \cos\left(\frac{\theta}{2}\right)$.

2) A mechanical system with 2 degrees of freedom has the Lagrangian

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}m(w_1^2x^2 + w_2^2y^2) + kxy$$

where m, w_1, w_2, k are constants. Find the parameter θ so that under the transformation

$$x = q_1\cos\ \theta - q_2\sin\ \theta,\ y = q_1\sin\ \theta + q_2\cos\ \theta$$

the Lagrangian in terms of q_1, q_2 will not contain the product term q_1q_2. Find the Lagrange's equations w.r.t. q_1 and q_2 independent of parameter θ. (2023)

3) A particle at a distance r from the centre of force moves under the influence of the central force $F = -\frac{k}{r^2}$, where k is a constant. Obtain the Lagrangian and derive the equations of motion. (2022)

4) Obtain the Lagrangian equation for the motion of a system of two particles of unequal masses connected by an inextensible string passing over a small smooth pulley. (2021)

5) Suppose the Lagrangian of a mechanical system is given by

$L = \frac{1}{2}m(ax^2 + 2bxy + cy^2) - \frac{1}{2}k(ax^2 + 2bxy + cy^2)$, Where $a, b, c, m(> 0), k(> 0)$ are constants and $b^2 \neq ac$. write down the Lagrangian equations of motion and identify the system. (2018)

6) Two uniform rods AB, AC each of mass m and length $2a$, are smoothly hinged together at A and move on a horizontal plane. At time t, the mass centre of the road is at the point (ξ, η) referred to fixed perpendicular axes Ox, Oy in the plane, and the rods make angles $\theta \pm \phi$ with Ox. Prove that the kinetic energy of the system is $m\left[\xi^2 + \eta^2 + \left(\frac{1}{3} + \sin^2\ \phi\right)a^2\theta^2 + \left(\frac{1}{3} + \cos^2\ \phi\right)a^2\phi^2\right]$. Also derive Lagrange's equation of motion for the system if an external force with components $[X, Y]$ along the axes acts at A. (2017)

7) A hoop with radius r is rolling, without slipping, down an inclined plane of length l and with angle of inclination ϕ. Assign appropriate generalized coordinate to the system. Determine the constraints, if any. Write down the Lagrangian equation for the system. Hence or otherwise determine the velocity of the hoop at the bottom of the inclined plane. (2016)

8) Two equal rod AB and BC, each of length l, smoothly jointed at B, are suspended from A and oscillate in a vertical plane through A. Show that that the periods of normal oscillation are $\frac{2\pi}{n}$ where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$ (2013)

9) Obtain the equations governing the motion of a spherical pendulum. (2012)

10) A uniform road of mass $3m$ and length $2l$ has its middle point fixed and a mass m is attached to one of its extremities. The rod, when in a horizontal position is set rotating about a vertical axis through its center with an angular velocity $\sqrt{\frac{28}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}(\sqrt{2} - 1)$. (2008)

11) A particle of mass m moves under the influence of gravity on the inner surface of the paraboloid of revolution $x^2 + y^2 = az$ which is assumed frictionless. Obtain the equation of motion show that it will describe a horizontal circle in the plane $z = h$, provided that it is given an angular velocity whose magnitude is $\omega = \sqrt{\frac{2g}{a}}$. (2004)

12) A uniform rod of length $2a$ which has one end attached to a fixed point by a light inextensible sting of length $\frac{5a}{12}$ performing small oscillations in a vertical plane about its position of equilibrium. Find the position at any time show that the periods of its principal oscillations are $2\pi\sqrt{\frac{5a}{3g}}$ and $\pi\sqrt{\frac{a}{3g}}$. (1992)

Hamiltonian problems

1) A particle of mass m moves in a force field of potential $V(r) = -\frac{k\cos\theta}{r^2}$, k is constant

Find the Hamiltonian and the Hamilton's equations in spherical polar coordinates (r, θ, ϕ).

2) Consider the Lagrangian $L = m\dot{x}\dot{y} - m\omega_0^2 xy$ where m and ω_0 are constants. Find the Hamiltonian and Hamilton's equations of motion. Identify the system.

3) A planet of mass m is revolving around the sun of mass M. The kinetic energy T and the potential energy V of the planet are given by $T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$ and $V = GMm\left(\frac{1}{2a} - \frac{1}{r}\right)$, where (r, θ) are the polar coordinates of the planet at time t, G is the gravitational constant and $2a$ is the major axis of the ellipse (the path of the planet). Find the Hamiltonian and the Hamilton equations of the planet's motion. (2023)

4) By writing down the Hamiltonian, find the equations of motion of a particle of mass m constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$ is a constant. The particle is subject to a force directed towards the origin and proportional to the distance r of the particle from the origin given by $\vec{F} = -k\vec{r}$, k is a constant (2020)

5) Using Hamilton's equation find the acceleration for a sphere rolling down a rough inclined plane, if x be the distance of the point of contact of the sphere from a fixed point on the plane. (2019)

6) The Hamiltonian of a mechanical system is given by $H = p_1q_1 - aq_1^2 + bq_2^2 - p_2q_2$ Where a, b are the constants. Solve the Hamiltonian equations and show that $\frac{p_2 - bq_2}{q_1}$ = constant. (2018)

7) Consider single free particle of mass m, moving in space under no forces. If the particle starts form the origin $t = 0$ at and reaches the position (x, y, z) at time τ, find the Hamilton's characteristic function S as a function of x, y, z, τ. (2016)

8) A Hamiltonian of a system with one degree of freedom has form

$H = \frac{p^2}{2\alpha} - bape^{-\alpha t} + \frac{b\alpha}{2}q^2e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{k}{2}q^2$ where α, b, k are constant, q is the generalized coordinate and p is the corresponding generalized momentum.

Find a Lagrangian corresponding too this Hamiltonian.

Find an equivalent Lagrangian that is not explicitly dependent on time. (2015)

9) Solve the plane pendulum problem using the Hamiltonian approach and show that H is a constant of motion. (2015)

10) Find the equation of motion of a compound pendulum using Hamilton's equations. (2014)

11) A sphere of radius a land mass m rolles down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equation. (2010)

12) A point mass m is placed on a frictionless plane that is tangent to the Earth's surface. Determine Hamilton's equations by taking x or θ as the generalized coordinate. (2007)

13) A particle of mass m is constrained to move on the surface of a cylinder. The particle is subject to a force directed towards the origin and proportional to the distance to of particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion. (2006)

14) Find the equation of motion for a particular of mass m which is constrained to move on the surface of a cone of semi-vertical angle α and which is subjected to a gravitational force. (2001)

Motion in Two Dimension Finite Force

1) A perfectly rough ball is at rest within a hollow cylindrical roller. The roller is drawn along a level path with uniform velocity V. Let a and b be the radii of the ball and the roller respectively. If $V^2 > \frac{27}{7} g(b-a)$, then show that the ball will roll completely round the inside of the rotter. (2023)

2) A circular cylinder of radius and radius a of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane thorough axes move in a circular pendulum of length $(b-a)\left(1+\frac{k^2}{a^2}\right)$. (2019)

3) A solid homogeneous sphere is rolling on the inside of a hollow sphere the centers being always in the same vertical plane. Show that the smaller sphere will make complete revolution if, when it is in lowest position, the pressure on it is greater than $\frac{34}{7}$ times its own weight. (1997)

4) The ends of a heavy rod of length $2a$ are rigidly attached to two light rings which can respectively slide on the thin smooth fixed horizontal and vertical wires O_x and O_y. The rod starts at an angle α to the horizon with an angular velocity $\sqrt{[3g(1-\sin\ \alpha)/2a]}$ and moved downwards. Show that it will strike the horizontal wire at the end of time $2\sqrt{a/(3g)}\log\left[\tan\left(\frac{\pi}{8}-\frac{\alpha}{4}\right)\cot\frac{\pi}{8}\right]$. (2011)

5) A uniform rod is placed with one end in contact with a horizontal table, and is then at an inclination α to the horizon and is allowed to fall when it becomes horizontal, show that its angular velocity is $\left(\frac{3g}{2a} sin\alpha\right)^{1/2}$ whether the plane is perfectly smooth or perfectly rough show

also that the end of the rod will not leave the plane in either case. An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon, to determine the motion.

6) If the earth supposed to be a uniform sphere had in a certain period contracted slightly so that its radius was less by $(1/n)th$ than before show that the length of the day would have shortened by $(48/n)$ hours.

7) A sphere of radius b, rolls without slipping down the cycloid $x = a(\theta + sin\theta); y = a(1 - cos\theta)$. It starts from rest with its centre on the horizontal line $y = 2a$. show that velocity v of its centre when at its lowest point is given by $v^2 = \frac{10}{7} g(2a - b)$.

8) A straight uniform rod can tum freely about one end O, hangs from O vertically. Find the least angular velocity with which it must begin to move so that it may perfom complete revolution in a vertical plane.

9) A perfectly rough circular horizontal board is capable of revolving freely round a vertical axis through the centre. A man whose weight is equal to that of the board walks on and around it at the edge. when he has completed the circuit, what will be his position in space.

10) A uniform circular disc is free to turn about a horizontal axis through its centre perp, to its plane. A particle of mass m is attached to a point in the edge of the disc. If the motion starts from the position in which radius to the particle makes an angle α with the upward vertical, find the angular velocity when m is in its lowest position. Take the mass of the disc as M

11) A disc rolls on the inside of a fixed hollow circular cylinder whose axis is horizontal, the plane of the disc being vertical and perpendicular to the axis of cylinder ; if , when in the lowest position, its centre is moving with a velocity $\left[\frac{8g}{3(a-b)}\right]^{1/2}$, show that the centre of the disc will describe an angle ϕ about the centre of the cylinder in time $\left[\frac{3(a-b)}{2g}\right]^{1/2} \cdot \log\ \tan\left(\frac{\pi}{4} + \frac{\phi}{4}\right)$.

Compound Pendulum

1) A rectangular plate swings in a vertical plane about one of its corners. If its period is one second, find the length of its diagonal. (2005)
2) A perfectly rough sphere of mass m and radius b, rests on the lowest point of a fixed spherical cavity a radius a. To the highest point of the movable sphere is attached a particle of mass m ' and the system is disturbed. Show that the oscillations are the same as of a simple pendulum of length $(a - b)\frac{4m' + \frac{7}{5}m}{m + m'\left(2 - \frac{a}{b}\right)}$.

3) A solid homogeneous cone of height h and vertical angle 2α oscillates about a honzontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{1}{5}h(4+\tan^2\ \alpha)$.

4) Find the time of oscillation of a compound pendulum consisting of a rod of mass m and length a, carrying at one end a sphere of mass m_1 and diameter $2b$, the other end of the rod being fixed.