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UPSC CSE Mathematics: Previous Year Questions: Analytical Geometry

2025

1) Find the equation of the cone whose vertex is the point $(1,1,0)$ and whose guiding curve is $\mathrm{y} = 0, \mathrm{x}^2 + \mathrm{z}^2 = 4$.
2) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and that passes through the curve $x^2 + y^2 = 16, z = 0$.
3) Find the shortest distance between the straight lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$
4) Find the equations of the spheres which pass through the circle $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0, 2x + y + z = 4$ and touch the plane $3x + 4y = 14$
5) Show that there is no tangent plane to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4z + 4 = 0$ that can be passed through the straight line $\frac{x+6}{2} = y + 3 = z + 1$

2024

1) Find the vertex of the cone $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$.
2) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.
3) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$.
4) Find the image of the line $x = 3 - 6t, \ y = 2t, \ z = 3 + 2t$ in the plane $3x + 4y - 5z + 26 = 0$.

2023

1) Show that the equation $2x^2 + 3y^2 - 8x + 6y - 12z + 11 = 0$ represents an elliptic paraboloid. Also find its principal axis and principal planes.
2) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in A, B, C respectively. Prove that the equation of the cone generated by the lines drawn from the origin O to meet the circle ABC is $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{b}{a} + \frac{a}{b}\right) = 0$.

3) Find the equation of the sphere through the circle
$x^2+y^2+z^2-4x-6y+2z-16=0; 3x+y+3z-4=0$ in the following two cases.
(i) the point $(1,0,-3)$ lies on the sphere.
(ii) the given circle is a great circle of the sphere.

4) Prove that the locus of a line which meets the lines $y=mx, z=c; y=-mx, z=-c$ and the circle $x^2+y^2=a^2, z=0$ is $c^2m^2(cy-mzx)^2+c^2(yz-cmx)^2=a^2m^2(z^2-c^2)^2$

2022

1) A variable plane passes through a fixed point (a, b, c) and meets the axes at points A, B and C respectively. Find the locus of the centre of the sphere passing through the points O, A, B and C, O being the origin.

2) If P, Q, R; P′, Q′, R′ are feet of the six normals drawn from a point to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, and the plane PQR is represented by $lx+my+nz=p$, show that the plane
$P'Q'R'$ is given by $\frac{x}{a^2 l}+\frac{y}{b^2 m}+\frac{z}{c^2 n}+\frac{1}{p}=0$

3) Find the equation of the sphere of smallest possible radius which touches the straight lines:
$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

4) If the plane $ux+vy+wz=0$ cuts the cone $ax^2+by^2+cz^2=0$ in perpendicular generators, then prove that $(b+c)u^2+(c+a)v^2+(a+b)w^2=0$

2021

1) Find the equation of the cylinder whose generators are parallel to the line
$x=-\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+2y^2=1, z=0$.

2) Show that the planes which cut the cone $ax^2+by^2+cz^2=0$ in perpendicular generators touch the cone $\frac{x^2}{b+c}+\frac{y^2}{c+a}+\frac{z^2}{a+b}=0$.

3) A sphere of constant radius r passes through the origin O and cuts the axes at the points A, B and C. Find the locus of the foot of the perpendicular drawn from O to the plane ABC.

4) Find the equation of the plane containing the lines
$$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7},$$
$$\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}.$$
Also find the point of intersection of the given lines.

2020

1) Find the equations of the tangent plane to the ellipsoid $2x^2+6y^2+3z^2=27$ which passes through the line $x-y-z=0=x-y+2z-9$

2) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4, \quad z = 2$

3) If the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then find the equations of the other two generators.

4) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$

2019

1) Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ intersect. Find the coordinates of the point of intersection and the equation of the plane containing them.

2) The plane $x + 2y + 3z = 12$ cuts the axes of coordinates in. A, B, C . Find the equations of the circle circumscribing the triangle ABC

3) Prove that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has vertex at $(2,4,1)$ in a rectangular hyperbola.

4) Prove that, in general, three normals can be drawn from a given point to the paraboloid $x^2 + y^2 = 2az$ but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$ then two of the three normals coincide.

5) Find the length of the normal chord through a point P of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and prove that if it is equal to $4PG_3$ where G_3 is the point where the normal chord through P meets xy plane, then P lies on the cone $\frac{x^2}{a^6}(2c^2 - a^2) + \frac{y^2}{b^6}(2c^2 - b^2) + \frac{z^2}{c^4} = 0$

2018

1) Find the projection of the straight line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1}$ on the plane $x + y + 2z = 6$

2) Find the shortest distance between the lines $a_1x + b_1y + c_1z + d_1 = 0, \quad a_2x + b_2y + c_2z + d_2 = 0$ and the z axis.

3) Find the equations to the generating lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point $(1,1,1)$

4) Find the equation of the sphere in xyz-plane passing through the points $(0,0,0), (0,1,-1), (-1,2,0)$ and $(1,2,3)$

5) Find the equation of cone with $(0,0,1)$ as the vertex and $2x^2 - y^2 = 4, z = 0$ as guiding curve.

6) Find the equation of the plane parallel to $3x - y + 3z = 8$ and passing through the point $(1,1,1)$

2017

1) Find the equation of the tangent at the point $(1,1,1)$ to the Conicoid $3x^2 - y^2 = 2z$.

2) Find the shortest distance between the skew lines:

$$\frac{x-3}{3} = \frac{8-y}{1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

3) A plane passes through a fixed point (a, b, c) and cuts the axes at the points A, B, C respectively. Find the locus of the center of the sphere which passes through the origin O and A, B, C

4) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Find the point of contact.

5) Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 - z^2 = a^2$. (2017)

6) Find the locus of the points of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$

7) Reduce the following equation to the standard form and hence determine the nature of the Conicoid: $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$.

2016

1) Find the equation of the sphere which passes though the circle $x^2 + y^2 = 4; z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3 .

2) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ and $y - mx = z = 0$. For what value of m will the two lines intersect?

3) Find the surface generated by a line which intersects the line $y = a = z$, $x + 3z = a = y + z$ and is parallel to the plane $x + y = 0$.

4) Show that the cone $3yz - 2zx - 2xy = 0$ has an infinite set of three mutually perpendicular generators. If $= \frac{y}{1} = \frac{z}{z}$ is a generator belonging to one such set, find the other two.

5) Find the locus of the point of intersection of three mutually perpendicular tangent planes to the Conicoid $ax^2 + by^2 + cz^2 = 1$

2015

1) Find what positive value of a, the plane $ax - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact.

2) If $6x = 3y = 2z$ represents one of the mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators.

3) Obtain the equation of the plane passing through the points $(2,3,1)$ and $(4, -5,3)$ parallel to x - axis

4) Verify if the lines: $\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$ are coplanar. If yes, find the equation of the plane in which they lie.

5) Two perpendicular tangent planes to the paraboloid $x^2+y^2=2z$ intersect in a straight line in the plane $x=0$. Obtain the curve to which this straight line touch.

2014

1) Examine whether the plane $x+y+z=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines

2) Find the co-ordinates of the points on the sphere $x^2+y^2+z^2-4x+2y=4$, the tangent planes at which are parallel to the plane $2x-y+2z=1$

3) Prove that equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$

4) Show that the lines drawn from the origin parallel to the normals to the central Conicoid $ax^2+by^2+cz^2=1$, at its points of intersection with the plane $lx+my+nz=p$ generate the cone $p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$

5) Find the equations of the two generating lines through any point $(a\cos\theta, b\sin\theta, 0)$ of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$, of the hyperboloid by the plane $z=0$

2013

1) Find the equation of plane which passes through points $(0,1,1)$ and $(2,0,-1)$ and is parallel to the line joining points $(-1, \ 1, \ -2), (3, \ -2, \ 4)$. Find also the distance between the line and the plane.

2) A sphere S has points $(0,1,0)$, $(3,-5,2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x-2y+4z+7=0$ as a great circle.

3) Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2+y^2+z^2=r^2$ from any point on the sphere $2(x^2+y^2+z^2)=3r^2$

4) A cone has for its guiding curve the circle $x^2+y^2+2ax+2by=0, z=0$ and passes through a fixed point $(0,0,c)$. If the section of the cone by the plane $y=0$ is a rectangular hyperbola, prove that vertex lies on the fixed circle $x^2+y^2+2ax+2by=0$, $2ax+2by+cz=0$

5) A variable generator meets two generators of the system through the extremities B and B^1 of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2c^2=1$ in P and P^1 . Prove that BP. $B^1P^1=a^2+c^2$

2012

1) Prove that two of the straight lines represented by the equation $x^3 + bx^2y + cxy^2 + y^3 = 0$ will be at right angles, if $b + c = -2$

2) A variable plane is parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C respectively. Prove that circle ABC lies on the cone $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$

3) Show that locus of a point from which three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z = 0$ is $x^2 + y^2 + 4z = 1$

2011

1) Find the equation of the straight line through the point $(3,1,2)$ to intersect the straight line $x + 4 = y + 1 = 2(z - 2)$ and parallel to the plane $4x + y + 5z = 0$

2) Show that the equation of the sphere which touches the sphere $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$ at the point $(1,2,-2)$ and the passes through the point $(-1,0,0)$ is $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$

3) Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ so that lines joining to , Q , R to origin are mutually perpendicular. Prove that plane PQR touches a fixed sphere

4) Show that the cone $yz + xz + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area

5) Show that generators through any one of the ends of an equi-conjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ are inclined to each other at an angle of 60^0 if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other.

2010

1) Show that the plane $x + y - 2z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle

2) Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact

3) Show that every sphere through the circle $x^2 + y^2 - 2ax + r^2 = 0, z = 0$ cuts orthogonally every sphere through the circle $x^2 + z^2 = r^2, y = 0$

4) Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid $\frac{x^2}{4} + y^2 - z^2 = 49$ passing through $(10,5,1)$ and $(14,2,-2)$

2009

1) A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ to meet two fixed lines $y=mx, z=c$ and $y=-mx, z=-c$. Find the locus of the line

2) Find the equation of the sphere having its center on the plane $4x-5y-z=3$ and passing through the circle $x^2+y^2+z^2-12x-3y+4z+8=0, 3x+4y-5z+3=0$

3) Prove that the normals from the point (α,β,γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone $\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$

2008

1) The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$; find the equation of the plane in its new position

2) Find the equations (in symmetric form) of the tangent line to the sphere $x^2+y^2+z^2+5x-7y+2z-8=0, 3x-2y+4z+3=0$ at the point $(-3,5,4)$.

3) A sphere S has points $(0,1,0)$, $(3,-5,2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x-2y+4z+7=0$ as a great circle

4) Show that the enveloping cylinders of the ellipsoid $a^2x^2+b^2y^2+c^2z^2=1$ with generators perpendicular to z - axis meet the plane $z=0$ in parabolas.

5) If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represent one of a set of three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$. find the equations of the other two.

2007

1) Find the equation of the sphere inscribed in the tetrahedron whose faces are $x=0, y=0, z=0$ and $2x+3y+6z=6$

2) Find the locus of the point which moves so that its distance from the plane $x+y-z=1$ is twice its distance from the line $x=-y=z$

3) Show that the spheres $x^2+y^2+z^2-x+z-2=0$ and $3x^2+3y^2-8x-10y+8z+14=0$ cut orthogonally. Find the center and radius of their common circle

4) A line with direction ratios 2, 7, -5 is drawn to intersect the lines $\frac{x}{3}=\frac{y-1}{2}=\frac{z-2}{4}$ and $\frac{x-11}{3}=\frac{y-5}{-1}=\frac{z}{1}$. Find the coordinate of the points of intersection and the length intercepted on it .

5) Show that the plane $2x-y+2z=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines

6) Show that the feet of the normals from the point $P(\alpha,\beta,\gamma), \beta\neq 0$ on the paraboloid $x^2+y^2=4z$ lie on the sphere $2\beta(x^2+y^2+z^2)-(\alpha^2+\beta^2)y-2\beta(2+\gamma)z=0$

2006

1) Show that the length of the shortest distance between the line $z = x\tan\alpha, y = 0$ and any tangent to the ellipse $x^2\sin^2\alpha + y^2 = a^2, z = 0$ is constant

2) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$

3) Show that the plane $ax + by + cz = 0$ cuts the cone $xy + yz + zx = 0$ in perpendicular lines, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

4) If the plane $lx + my + nz = p$ passes through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Prove that $a^2l^2 + b^2m^2 + c^2n^2 = 3p^2$

2005

1) A square $ABCD$ having each diagonal AC and BD of length $2a$, is folded along the diagonal AC so that the planes DAC and BAC are at right angle. Find the shortest distance between AB and DC

2) A plane is drawn through the line $x + y = 1, z = 0$ to make an angle $\sin^{-1}\left(\frac{1}{3}\right)$ with plane $x + y + z = 5$. Show that two such planes can be drawn. Find their equations and the angle between them.

3) Show that the locus of the centers of sphere of a co-axial system is a straight line.

4) Obtain the equation of a right circular cylinder on the circle through the points $(a, 0, 0), (0, b, 0), (0, 0, c)$ as the guiding curve.

5) Reduce the following equation to canonical form and determine which surface is represented by it: $2x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y - 6z + 2 = 0$

2004

1) Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$, which are parallel to the plane $2x + y - z = 4$

2) Prove that the locus of a line which meets the lines $y = \pm mx, z = \pm c$ and the circle $x^2 + y^2 = a^2, z = 0$ is $c^2m^2(cy - mzx)^2 + c^2(yz - cmx)^2 = a^2m^2(z - c^2)^2$

3) Prove that the lines of intersection of pairs of tangent planes to $ax^2 + by^2 + cz^2 = 0$ which touch along perpendicular generators lie on the cone $a^2(b + c)x^2 + b^2(c + a)y^2 + c^2(a + b)z^2 = 0$

4) Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ through the point (α, β, γ). Prove that the perpendiculars to them through the origin generate the cone $(\alpha x + \beta y + \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2$

2003

1) A variable plane remains at a constant distance unity from the point $(1,0,0)$ and cuts the coordinate axes at A, B, and C, Find the locus of the centre of the sphere passing through the origin and the point and the point A, B and C.

2) Find the equation of the two straight lines through the point $(1,1,1)$ that intersect the line

$x - 4 = 2(y - 4) = 2(z - 1)$ at an angle of $60°$

3) Find the volume of the tetrahedron formed by the four planes $lx + my + nz = p, lx + my = 0,\ \ my + nz = 0$ and $nz + lx = 0$

4) A sphere of constant radius r passes through the origin O and cuts the co-ordinate axes at A, B and C. Find the locus of the foot of the perpendicular from O to the plane ABC.

5) Find the equations of the lines of intersection of the plane $x + 7y - 5z = 0$ and the cone

$$3xy + 14zx - 30xy = 0$$

6) Find the equations of the line of shortest distance between the lines: $y + z = 1, x = 0$ and $x - z = 1, y = 0$ as the intersection of two planes

2002

1) Find the co-ordinates of the center of the sphere inscribed in the tetrahedron formed by the plane $x = 0, y = 0, z = 0$ and $x + y + z = a$

2) Consider a rectangular parallelepiped with edges a, b and c. Obtain the shortest distance between one of its diagonals and an edge which does not intersect this diagonal

3) Show that the feet of the six normals drawn from any point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie on the cone $\frac{a^2(b^2-c^2)\alpha}{x} + \frac{b^2(c^2-a^2)\beta}{y} + \frac{c^2(a^2-b^2)\gamma}{z} = 0$

4) A variable plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ parallel to the plane meets the co-ordinate axes at A, B and C. Show that the circle ABC lies on the conic $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$

2001

1) Find the shortest distance between the axis of z and the lines

$$ax + by + cz + d = 0, \qquad a'x + b'y + c'z + d' = 0$$

2) Find the equation of the circle circumscribing the triangle formed by the points $(a, 0,0), (0, \mathrm{b}, 0), (0,0, \mathrm{c})$. Obtain also the coordinates of the center of the circle.

3) Find the locus of equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

4) Prove that $5x^2 + 5y^2 + 8z^2 + 8yz + 8zx - 2xy + 12x - 12y + 6 = 0$ represents a cylinder whose cross-section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$