SuccessClap
Best Coaching for UPSC Mathematics

Note: Solutions of all the questions are available in sequence in Crash Course. There is very less time for Mains exam and it is not possible to solve all problems, hence it is advisable to go through the solutions (Quick glance) of all these important questions at once. All topics, Important questions and important PYQ questions are covered.

Important Questions for UPSC Maths 2026

FLUID DYNAMICS

1) Show that for an incompressible steady flow with constant viscosity, the velocity components
$$u(y)=\left(\frac{U}{h}\right)y-\frac{hy}{2\mu}\frac{dp}{dx}\left(1-\frac{y}{h}\right)$$
$v=0=w$, with $p=p(x)$, satisfy the equation of motion in the absence of body force. Given that U, h and $\frac{dp}{dx}$ are constants.

2) Determine under what conditions the velocity field $u=c(x^2-y^2), v=-2cxy, w=0$ is a solution to the Navier-Stokes momentum equations. Assuming that the conditions are met, determine the resulting pressure distribution, when z ' is up and the external body forces are $B_x=0=B_y, B_z=-g$.

3) In a fluid motion, there is a source of strength $2m$ placed at $z=2$ and two sinks of strength m are placed at $z=2+i$ and $z=2-i$. Find the streamlines.

4) Two-point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-\frac{k}{2}$ is situated at the origin. Show that the fluid motion is stationary and also find the equations of streamlines. If the streamlines, which pass through the stagnation points, meet the x-axis at $(\pm b, 0)$, then show that $3\sqrt{3}(b^2-a^2)^2=16a^3b$

5) Verify that $w=ik\log\{(z-ia)/(z+ia)\}$ is the complex potential of a steady flow of fluid about a circular cylinder, where the plane $y=0$ is a rigid boundary. Find also the force exerted by the fluid on unit length of the cylinder.

6) Show that for the complex potential $\tan^{-1} z$, the streamlines and equipotential curves are circles. Find the velocity at any point and check the singularities at $z=\pm i$.

7) Two sources of strength $\frac{m}{2}$ are placed at the points $(\pm a, 0)$. Show that at any point on the circle $x^2 + y^2 = a^2$ the velocity is parallel to the y axis and is inversely proportional to y.

8) For an incompressible fluid flow, two components of velocity (u, v, w) are given by $u = x^2 + 2y^2 + 3z^2$ $v = x^2y - y^2z + zx$. Determine the third component w so that they satisfy the equation of continuity. Also, find the z-component of acceleration.

9) For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2y - xy^2 + \frac{1}{3}(x^3 - y^3)$.Determine the velocity components along the directions x and y. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not.

10) Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$ possess vorticity, where $\vec{q}(u, v, w)$ is the velocity in the Cartesian frame $\vec{r}(x, y, z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9, z = 0$?

11) A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{\imath}$ Show that he velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\ \theta$, where $OP = r$ and θ is the angle which $\overrightarrow{OP}$ makes with the direction $\vec{\imath}$. Find the deferential equation of the streamlines and show that the lie on the surfaces $Ur^2\sin^2\ \theta - 2m\cos\ \theta =$ constant.

12) The space between two concentric spherical shells of radii $a, b (a < b)$ is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y - direction, then show has the initial motion of the liquid is given by velocity potential $\phi = \frac{\left\{a^3U\left(1+\frac{1}{2}b^3r^{-3}\right)x - b^3V\left(1+\frac{1}{2}a^3r^{-3}\right)y\right\}}{(b^3-a^3)}$, where $r^2 = x^2 + y^2 + z^2$, the coordinate being rectangular. Evaluate the velocity at any point of the liquid.

13) Consider a uniform flow U_0 in the positive x-direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points.

14) In an axis symmetric motion, show that stream function exists due to equation of continuity. Express the velocity components of the stream function. Find the equation satisfied by the stream function if the flow is irrotational.

15) Find Navier-Stokes equation for steady laminar flow of a viscous incompressible fluid between two infinite parallel plates.

16) If n rectilinear vortices of the same strength K are symmetrically arranged as generators of a circle cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2a^2}{(n-1)\mathrm{K}}$. Find the velocity at any point of the liquid.

17) A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

18) Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q} \to 0$ as $r \to \infty$, find the surfaces of constant speed.

19) An infinite row of the equidistance rectilinear vortices are at distance a apart. The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function.

20) In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solution of Laplace's equation.

21) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distances from its axis, show that the path of each vortex is given by the equation $(r^2\sin^2 \theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2\sin^2 \theta$, θ being measured from the line through the centre perpendicular to the joint of the vortices.

22) Let the fluid fills the region $x \geq 0$ (right half of $2d$ plane). Let a source α be $(0, y_1)$ and equal sink at $(0, y_2), y_1 > y_2$. Let the pressure be same as pressure infinity i.e., p_0. Show that the resultant pressure on the boundary (y-axis) is $\pi\rho\alpha^2(y_1 - y_2)^2/2y_1y_2(y_1 + y_2), \rho$ being the density of the fluid.

23) State the conditions under which Euler's equation of motion can be integrated show that $-\frac{\partial\phi}{\partial t} + \frac{1}{2}q^2 + V\int \frac{dp}{\rho} = F(t)$ where the symbols have their usual meaning.

24) Prove that $\left(v\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2\psi = \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}$ where V is the kinematic viscosity of the fluid and ψ is the stream function for a two-dimensional motion of a viscous fluid.

25) Show that the velocity distribution in axial flow of viscous incompressible fluid along a pipe of annular cross-section radii $r_1 < r_2$, is given by $\omega(r) = \frac{1}{4\mu}\frac{dp}{dz}\left\{r^2 - r_1^2 + \frac{r_2^2-r_1^2}{\log\left(\frac{r_2}{r_1}\right)}\log\left(\frac{r}{r_2}\right)\right\}$.

26) Prove that the equation of motion of a homogeneous inviscid liquid moving under forces arising from a potential V may be written in the form

$$\frac{\partial\vec{q}}{\partial t} - \vec{q} \times \vec{\zeta} = -\vec{\nabla}\left(\frac{p}{\rho} + \frac{1}{2}\vec{q}^2 + V\right)$$

where $\vec{q}, t, \vec{\zeta}, p, \rho$ respectively stand for velocity vector, time, vorticity vector, pressure, density, and $\vec{\nabla}()$ the gradient operator.

If the velocity $\vec{q}$, referred to cylindrical polar coordinates (r, θ, z), is given by

$$\vec{q} = \begin{cases} \left[0, \frac{1}{2}\omega r, 0\right] & (0 \le r \le a) \\ \left[0, \frac{1}{2}\frac{\omega a^2}{r}, 0\right] & (r \ge a) \end{cases}$$

where ω is a constant, prove that the vorticity is given by $\vec{\zeta} = \begin{cases} [0,0,\omega] & (0 \le r \le a) \\ [0,0,0] & (r \ge a) \end{cases}$

27) A long infinite cylinder of radius a is placed in a uniform stream such that its axis lies perpendicular to the stream. Besides, a circulation round the cylinder is produced by a uniform line vortex through the origin. If the uniform stream velocity is $-U\hat{\imath}$ and the circulation is $2\pi k$, then find out the complex velocity potential. Show, by using the theorem of Blasius, that the cylinder experiences an uplifting force.

28) There is a doublet at $(c, 0)$ in a 2-dimensional flow. A cylinder of radius a $(\mathrm{a} < c)$ with z-axis as axis of the cylinder was introduced into the flow. Find the complex potential and image system for the flow.

29) Consider 2-dimensional Navier-Stokes equations of a steady fluid flow.Show that there exists a stream function $\Psi(\mathrm{x}, \mathrm{y})$ for such a flow. Find the equation satisfied by $\Psi(x, y)$.

30) In a two-dimensional fluid flow, the velocity components are given by $u = x - ay$ and $v = -ax - y$, where a is constant. Show that the velocity potential exists for this flow and determine the appropriate velocity potential. Also, determine the corresponding stream function that would represent the flow.

31) In the case of two-dimensional motion of a liquid streaming past a fixed circular disc, the velocity at infinity is u in a fixed direction, where is u is a variable. Show that the maximum value of the velocity at any point of the fluid is $2u$. Prove that the force necessary to hold the disc is, $2m\dot{u}$ where m is the mass of the liquid displaced by the disc.

32) With Usual notations, show that ϕ and ψ for a uniform flow past a stationary cylinder are given by

$$\phi = U\cos\,\theta\left(r + \frac{a^2}{r}\right)$$

$$\psi = U\sin\,\theta\left(r - \frac{a^2}{r}\right)$$

33) For a steady Poiseuille flow through a tube of uniform circular cross-section show that

$$w(R) = \frac{1}{4}\left(\frac{p}{\mu}\right)(a^2 - R^2)$$

34) A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being Π. The sphere is suddenly annihilated. Show that the pressure at a distancer from the centre of the sphere immediately falls to $\Pi\left(1-\frac{a}{r}\right)$

35) Prove that the image system for a source outside a circle consists of an equal source at the inverse point and an equal sink at the center of the circle.

36) State the conditions under which the equations of motion can be integrated. Obtain Bernoulli's equations for the steady irrotational motion of an incompressible liquid.

37) In a 2-dimensional flow there are sources at $(a,0),(-a,0)$ and sinks at $(0,a),(0,-a)$, all are of equal strength. Determine the stream function and show that the circle through these four points is a streamline.

38) A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from the origin where the velocity isu_0 to become $\rho=\rho_0\phi(vt-x)$. Prove that the velocity u (at time t and distance x from the origin) is given by $v+\frac{(u_0-v)\phi(vt)}{\phi(vt-x)}$. where v is the velocity at that end of the tube and a is a constant.

39) Show that in a two- dimensional incompressible steady flow field the equation of continuity is satisfied with the velocity components in rectangular coordinates given by $u(x,y)=\frac{k(x^2-y^2)}{(x^2+y^2)^2}$, $v(x,y)=\frac{2kxy}{(x^2+y^2)^2}$, whether k is an arbitrary constant.

40) Determine the constants l, m and n in order that the velocity

$\mathbf{q}=\{(x+lr)\mathbf{i}+(y+mr)\mathbf{j}+(z+nr)\mathbf{k}\}/\{r(x+r)\}$

where $r=(x^2+y^2+z^2)^{1/2}$ may satisfy the equation of continuity for a liquid.

41) Liquid flows through a pipe whose surface is the surface of revolution of the curve $y=a+(kx^2/a)$ about the x – axis ($-a\le x\le a$). If the liquid enters at the end x = -a of the pipe with velocity V, show that the time taken by a liquid particle to transverse the entire length of the pipe from x = -a to x =a is $\{2a/V(1+k)^2\}\{1+2k/3)+(k^2/5)\}$. Assume that k is so small that flow remains appreciably one dimensional throughout.

42) Determine the restrictions on f_1, f_2, f_3 if $(x^2/a^2)f_1(t)+(y^2/b^2)f_2(t)+(z^2/c^2)f_3(t)=1$ is a possible boundary surface of a liquid.

43) Show that the ellipsoid $\frac{x^2}{a^2e^{-t}\cos\left(t+\frac{\pi}{4}\right)}+\frac{y^2}{b^2e^t\sin(1+\sin\left(t+\frac{\pi}{4}\right)}+\frac{z^2}{c^2\sec 2t}=1$ is a possible form of boundary surface of a liquid for any time t and determine the velocity q of any particle on this boundary. Also prove that the equation of continuity is satisfied.

44) Find the streamlines and paths of the particles when $u=x/(1+t)$, $v=y/(1+t)$, $w=z/(1+t)$.

45) Prove that if the speed is everywhere the same, the streamlines are straight lines.

46) For an incompressible homogeneous fluid at the point (x,y,z) the velocity distribution is given by $u = -(c^2y/r^2)$, $v = c^2x/ r^2$, $w = 0$, where r denotes the distance from the z axis. Show that it is a possible motion and determine the surface which is orthogonal to streamlines.

47) Determine the streamlines and the path lines of the particle when the components of the velocity field are given by $u = x/(1+t)$, $v = y/(2+t)$ and $w = z/(3+t)$. Also state the condition for which the streamlines are identical with path lines.

48) Test whether the motion specified by $q = \frac{k^2(xj-yi)}{x^2+y^2}$ (k = constant), is a possible motion for an incompressible fluid. If so, determine the equation of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.

49) At a point in an incompressible fluid having spherical polar co-ordinates (r,θ,ϕ), the velocity components are $[2Mr^{-3}\cos\theta, Mr^{-3}\sin\theta, 0]$, where M is a constant. Show the velocity is of the potential kind. Find the velocity potential and the equations of the stream lines.

50) Show that $u = -\frac{2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational. Find the vorticity vector.

51) Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two-dimensional fluid. Show that the streamlines are time 't' are the curves $(x-t)^2-(y-t)^2 =$ constant, and that the paths of the fluid particles have the equations $\log(x-y) = (1/2) \times \{(x+y) - a(x-y)^{-1}\} + b$, where a, b are constants.

52) If the velocity of an incompressible fluid at the point (x,y,z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$ prove that the liquid motion is possible and that the velocity potential is $(\cos\theta)/r^2$. Also determine the streamlines.

53) If velocity distribution of an incompressible fluid at point (x,y,z) is given by $\{3xz/r^5, 3yz/r^5, (kz^2-r^2)/r^5\}$, determine the parameter k such that it is a possible motion. Hence find its velocity potential.

54) Show that if the velocity potential of an irrotational fluid motion is equal to $A(x^2+y^2+z^2)^{-3/2} z \tan^{-1}(y/x)$ the lines of flow will be on the series of the surfaces $x^2+y^2+z^2 = c^{2/3}(x^2+y^2)^{2/3}$.

55) Prove that the liquid motion is possible when velocity at (x,y,z) is given by $u = (3x^2-r^2)/r^5$, $v = 3xy/r^5$, $w = 3xz/r^5$, where $r^2 = x^2+y^2+z^2$, and the streamlines are the intersection of the surfaces $(x^2+y^2+z^2)^3 = c(y^2+z^2)^2$ by the planes passing through OX. State if the motion is irrotational giving reasons for your answer.

56) Show that the velocity potential $\phi = (a/2) \times (x^2+y^2 - 2z^2)$ satisfies the Laplace equation. Also determine the streamlines.

57) A sphere of radius R, whose centre is at rest, vibrates radically in an infinite incompressible fluid of density ρ, where is at rest at infinity. If the pressure at infinity is $\prod$. Show that the pressure at the surface of the sphere at time t is $\prod + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}$

If $R = a(2 + \cos n\,nt)$, show that, to prevent cavitation in the fluid, $\prod$ must not be less than $3\rho a^2 n^2$.

58) An infinite mass of homogenous incompressible fluid is at uniform pressure $\prod$ and contains a spherical cavity of radius a, filled with a gas at pressure $m\prod$; Prove that if the inertia of the gas be neglected, and Boyle's law be supposed to hold throughout the ensuring motion, the radius of the sphere will oscillate between the values a and na, where n is determined by the equation $1 + 3m\log n - n^3 = 0$.

If m be nearly equal to 1. The time of an oscillation will be $2\pi\sqrt{(a^2\rho/3\pi}$, ρ being the density of the fluid.

59) A mass of gravitating fluid is at rest under its own attraction only, the free surface being a sphere of radius b and the inner surface a rigid concentric shell of radius a. Show that if the shell suddenly disappears, the initial pressure at any point of the fluid at distance r from the center is $\frac{1}{2}\pi\gamma\rho^2(b-a)(r-a)\left(\frac{a+b}{r}+1\right)$. *(Not Very Important)*

60) Liquid is contained between two parallel planes; the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated: prove that if $\prod$ be the pressure at the outer surface, the initial pressure at any point on the liquid distance r from the centre is

$$\prod\frac{\log r - \log b}{\log a - \log b}.$$

61) A mass of liquid of density ρ whose external surface is a long circular cylinder of radius a which is subject to a constant pressure $\prod$, surrounds a coaxial long circular cylinder of radius b. The internal cylinder is suddenly destroyed; show that if v is the velocity at the internal surface, when the radius is r, then $v^2 = \frac{2\prod(b^2-r^2)}{\rho r^2\log(r^2+a^2-b^2)/r^2}$. *(Not Very Important)*

62) Show that the velocity vector q is everywhere tangent to lines in the xy plane along which $\psi(x,y) = \text{const}$.

63) A steady inviscid incompressible fluid has a velocity u = fx, $v = -fv$, $w = 0$, where if is a constant. Derive an expression for the pressure field $p(x,y,z)$ if the pressure $p(0,0,0) = p_0$ and $F = -giz$.

64) Show that if the velocity field $u(x,y) = \frac{B(x^2-y^2)}{(x^2+y^2)^2}$, $v(x,y) = \frac{2Bxy}{(x^2+y^2)^2}$ $w(x,y) = 0$ satisfies the equations of motion for inviscid incompressible flow, then determine the pressure associated with this velocity field, B being a constant.

65) An infinite mass of fluid is acted on by a force $u/r^{3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = c$ in it, show that the cavity will be filled up after an interval of time $(2/5u)^{1/2}C^{5/4}$.

66) A stream is rushing from a boiler through a conical pipe, the diameter of the ends of which are D and d; if V and v be the corresponding velocities of the stream and if the motion be supposed to be that of the divergence from the vertex of the cone, prove that

$$v/V = (D^2/d^2)e^{(v^2-V^2)/2k}$$

Where k is the pressure divided by the density and supposed constant.

67) A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth $(s^2/2g) \times (1/A^2 - 1/B^2)$ Below the pipe, s being the delivery per second

68) A mass of homogeneous liquid is moving so that the velocity at any point is proportional to the time and that the pressure is given by $p/\rho = \mu xyz - (t^2/2) \times (y^2z^2 + z^2x^2 + x^2y^2)$.
Prove that this motion may have been generated from rest by natural forces independent of the time and show that, if the direction of motion at every point coincides with the direction of the acting force, each particle of the liquid describes a curve which is the intersection of two hyperbolic cylinders. *(Not Very Important)*

69) A quantity of liquid occupies a length $2l$ of a straight tube of uniform small bore under the action of a force to a point in the tube varying as a distance from that point. Determine the pressure at any point. OR
A quantity of liquid occupies a length $2l$ of a straight tube of uniform bore under the action of force which is equal to μx to a point O in the tube, where x is the distance from O. Find the motion and show that if z be the distance of the nearer free surface from O, pressure at any point is given by $p/\rho = -(\mu/2) \times (x^2 - z^2) + \mu(x-z)(z+l)$. *(Not Very Important)*

70) In irrotational motion in two dimensions, prove that $(\partial q/\partial x)^2 + (\partial q/\partial y)^2 = q\nabla^2 q$.

71) Show that the velocity potential $\phi = \frac{1}{2} log \frac{(x+a)^2+y^2}{(x-a)^2+y^2}$ gives a possible motion. Determine the streamlines and show also that the curves of equal speed are the ovals of Cassini given by rr' = const

72) A two – dimensional flow field is given by $\psi = xy$. (a) Show that the flow is irrotational. (b) Find the velocity potential. (c) Verify that ψ and ϕ satisfy the Laplace equation. (d) Find the streamlines and potential lines.

73) What arrangement of sources and sinks will give rise to the function $w = \log(z - a^2/z)$. Draw a rough sketch of the streamlines. Prove that two of the streamlines subdivide into the circle r = a and axis of y.

74) Two sources, each of strength m are placed at the points (-a,0),(a,0) and a sink of strength 2m at the origin. Show that the streamlines are the curves $(x^2+y^2)^2 = a^2 (x^2-y^2+ \lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2ma^2)/ (r_1 r_2 r_3)$ where r_1, r_2, r_3 are the distances of the points from the sources and the sink.

75) Find the stream function of the two – dimensional motion due to two equal sources and an equal sink situated midway between them.

76) Between the fixed boundaries $\Theta = \frac{\pi}{6}$ and $\theta = -\frac{\pi}{6}$ there is a two – dimensional liquid motion due to a source at the point (r = c, $\Theta = \alpha$) and a sink at the origin absorbing water at the same rate as the source produces. Find the stream function and show that one of the stream lines is a part of the curve $r^3 \sin 3\alpha = c^3 \sin 3\Theta$.

77) Between two fixed boundaries $\theta = \frac{\pi}{4}$ and $\theta = -\frac{\pi}{4}$, there is two – dimensional liquid motion due to a source of strength m at the point (r = a, Θ = 0) and an equal sink at the point (r =b, Θ =0). Show that the stream function is $-m \tan^{-1} \frac{r^4(a^4-b^4)\sin 4\theta}{r^8 - r^4(a^4+b^4)\cos 4\theta + a^4 b^4}$ and show that the velocity at (r,θ) is $\frac{4m(a^4-b^4)r^3}{(r^8-2a^4r^4\cos 4\theta+a^8)^{\frac{1}{2}}(r^8-2b^4r^4\cos 4\theta+b^8)^{1/2}}$.

78) Prove that for liquid circulating irrotationally in part of the fluid between two non – intersecting circles the curves of constant velocity are Cassini's Ovals.

79) Use the method of images to prove that if there be a source m at the point z_0 in a fluid bounded by the lines Θ=0 and $\Theta = \pi/3$, the solution is $\varphi + i\psi = -m\log\{(z^3 - z_0^3)(z^3 - z'^3_0)\}$ where $z_0 = x_0 + iy_0$ and $z'_0 = x_0 - iy_0$.

80) If fluid fills the region of space on the positive side of the x – axis, which is a rigid boundary and if there be a source m at the point (0,a) and an equal sink at (0,b) and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\pi\rho m^2 (a-b)^2 / 2ab(a+b)$, where ρ is the density of the fluid.

81) Parallel line sources (perpendicular to xy – plane) of equal strength m are parallel to the points z = nia where n =…., -2, -1, 0, 1, 2, …. Prove that the complex potential is w = -m log sinh $(\pi z/a)$. Hence, show that the complex potential for two dimensional doublets (line doublets), with their axes parallel to the x – axis, of strength μ at the same points is given by w = $\mu \coth(\pi z/a)$.

82) In the case of the motion of liquid in a part of a plane bounded by a straight line due to a source in the plane, prove that if m ρ is the mass of fluid (of density ρ) generated at the source per unit of time the pressure on the length 2l of the boundary immediately opposite to the source is less than that on an equal length at a great distance by $\frac{1}{\rho}\frac{m^2\rho}{\pi^2}\left[\frac{1}{c}\tan^{-1}\frac{1}{c} - \frac{l}{l^2+c^2}\right]$, where c is the distance of source to the boundary.

83) A source and sink of equal strength are placed at the points $(\pm a/2, 0)$ within a fixed circular boundary $x^2+y^2 = a^2$. Show that the streamlines are given by $(r^2-a^2/4)(r^2-4a^2) - 4a^2y^2 = ky(r^2-a^2)$.

84) Liquid of density ρ is flowing in two dimensions between the oval curves $r_1r_2 = a^2$ and $r_1r_2 = b^2$ where r_1, r_2 are the distances measured from two fixed points. If the motion is irrotational and quantity m per unit time crosses any line joining the bounding curves, then prove that the kinetic energy is $(\pi\rho m^2)/\log(b/a)$.

85) Incompressible fluid of density ρ is contained between two co-axial circular cylinders of radii a and b (a < b), and between two rigid planes perpendicular to the axis at a distance l apart. The cylinders are at rest and the fluid is circulating in irrotational motion, its velocity V at the surface of the inner cylinder. Prove that the kinetic energy is $\pi\rho l a^2 V^2 \log(b/a)$.

86) The space between two infinitely long coaxial cylinders of radii a and b (> a) respectively is filled with homogeneous liquid of density ρ. The inner cylinder is suddenly moved with velocity U perpendicular to the axis, the outer one being kept fixed. Show that the resultant impulsive pressure on a length l of the inner cylinder is $\pi\rho a^2 l \frac{b^2+a^2}{b^2-a^2}U$.

87) Prove that the necessary and sufficient condition that the vortex lines may be at right angles to the stream lines are u,v,w = $\mu\left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right)$ *where* μ, φ are functions of x, y,z,l. **OR** Find the necessary and sufficient condition that vortex lines may be at right angles to the streamlines.