SuccessClap

Best Coaching for UPSC Mathematics

UPSC CSE Mathematics: Previous Year Questions: ODE

2025

1) Solve $\left(1-y^2+\frac{y^4}{x^2}\right)\left(\frac{dy}{dx}\right)^2-2\frac{y}{x}\frac{dy}{dx}+\frac{y^2}{x^2}=0$

2) Form the differential equation of all ellipses whose axes coincide with coordinate axes.

3) If $\mathrm{F(s)}$ and $\mathrm{G(s)}$ are Laplace transforms of $\mathrm{f(t)}$ and $\mathrm{g(t)}$ respectively, then prove that $\mathcal{L}\left(\int_0^t \mathrm{f(x)g(t-x)dx}\right)=\mathrm{F(s)G(s)}$. Using this result, solve the equation
$$\mathrm{y(t)=t+\int_0^t y(x)\sin(t-x)dx}.$$

4) Find the general solution and singular solution of the differential equation $\left(1+\frac{dy}{dx}\right)^3=\frac{27}{8a}(x+y)\left(1-\frac{dy}{dx}\right)^3$.

5) Find the complete solutions of $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x\log x$

6) Solve the differential equation $(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(1+x)e^x$ by the method of variation of parameters.

2024

1) Find the orthogonal trajectories of the family of curves $r=c(\sec\theta+\tan\theta)$, where c is a parameter.

2) Solve the integral equation $y(t)=\cos t+\int_0^t y(x)\cos(t-x)dx$ using Laplace transform.

3) Find the second solution of the differential equation $xy''+(x-1)y'-y=0$ using $u(x)=-e^{-x}$ as one of the solutions.

4) Find the general solution of the differential equation $x^2y''-2xy'+2y=x^3\sin x$ by the method of variation of parameters.

5) State uniqueness theorem for the existence of unique solution of the initial value problem $\frac{dy}{dx}=f(x,y), y(x_0)=y_0$ in the rectangular region $R: |x-x_0|\le a, |y-y_0|\le b$. Test the existence and uniqueness of the solution of the initial value problem $\frac{dy}{dx}=2\sqrt{y},\quad y(1)=0$, in a suitable rectangle R. If more than one solution exists, then find all the solutions.

6) Using Laplace transform, solve the initial value problem

$$y'' + 2y' + 5y = \delta(t-2),\ y(0) = 0,\ y'(0) = 0$$

where $\delta(t-2)$ denotes the Dirac delta function.

2023

1) Obtain the solution of the initial-value problem $\frac{dy}{dx} - 2xy = 2,\ y(0) = 1$ in the form $y = e^{x^2}[1 + \sqrt{\pi}\text{erf}\,(x)]$

2) Given that $L\{f(t);p\} = F(p)$. Show that $\int_0^\infty \frac{f(t)}{t}dt = \int_0^\infty F(x)dx$. Hence evaluate the integral $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}dt$

3) Solve the differential equation: $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos\ x.$

4) Find the solution of differential equation : $\frac{dy}{dx} = -\frac{2xy^3+2}{3x^2y^2+8e^{4y}}$

5) Reduce the equation $x^2p^2 + y(2x+y)p + y^2 = 0$ to Clairaut's form by the substitution $y = u$ and $xy = v$. Hence solve the equation and show that $y + 4x = 0$ is a singular solution of the differential equation.

6) If the tangent to a curve makes a constant angle θ with a fixed line, then prove that the ratio of radius of torsion to radius of curvature is proportional to $\tan\ \theta$. Further prove that if this ratio is constant, then the tangent makes a constant angle with a fixed direction.

7) Solve the following initial value problem by using Laplace transform technique $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y(t) = f(t)$ $\quad y(0) = 1, y'(0) = 0$ and $f(t)$ is a given function of t.

2022

1) Show that the general solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{Py} = \mathrm{Q}$ can be written in the form $\mathrm{y} = \frac{\mathrm{Q}}{\mathrm{P}} - \mathrm{e}^{-\int \mathrm{Pdx}}\left\{\mathrm{C} + \int \mathrm{e}^{\int \mathrm{Pdx}}\mathrm{d}\left(\frac{\mathrm{Q}}{\mathrm{P}}\right)\right\}$, where P, Q are non-zero functions of x and C, an arbitrary constant.

2) Show that the orthogonal trajectories of the system of parabolas $\mathrm{x}^2 = 4\mathrm{a}(\mathrm{y}+\mathrm{a})$ belong to the same system.

3) Solve the following differential equation by using the method of variation of parameters: $(x^2-1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = (x^2-1)^2$, given that $y = x$ is one solution of the reduced equation.

4) Solve the following initial value problem by using Laplace's transformation $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dt}^2} - 3\frac{\mathrm{dy}}{\mathrm{dt}} + 2\mathrm{y} = \mathrm{h}(\mathrm{t})$, where $h(t) = \begin{cases} 2, & 0 < t < 4, \\ 0, & t > 4, \end{cases}$ $y(0) = 0, y'(0) = 0$

5) Find the general and singular solutions of the differential equation:

$(\mathrm{x}^2 - \mathrm{a}^2)\mathrm{p}^2 - 2\mathrm{xyp} + \mathrm{y}^2 + \mathrm{a}^2 = 0$, where $\mathrm{p} = \frac{\mathrm{dy}}{\mathrm{dx}}$. Also give the geometric relation between the general and singular solutions.

6) Solve the following differential equation : $(3x+2)^2\frac{d^2y}{dx^2}+5(3x+2)\frac{dy}{dx}-3y=x^2+x+1$

2021

1) Solve the differential equation $\frac{d^2y}{dx^2}+2y=x^2e^{3x}+e^x\cos\ 2x$

2) Solve the initial value problem using Laplace transform method:
$$\frac{d^2y}{dx^2}+4y=e^{-2x}\sin\ 2x; y(0)=y'(0)=0$$

3) Solve $\frac{d^2y}{dx^2}+(\tan x-3\cos x)\frac{dy}{dx}+2y\cos^2 x=\cos^4 x$completely by demonstrating all the steps involved.

4) Find all possible solutions of the differential equation: $y^2\log\ y=xy\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2$

5) Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1;\quad a>b>0$ are constants and λ is a parameter. Show that the given family of curves is self-orthogonal.

6) Find the general solution of the differential equation: $x^2\frac{d^2y}{dx^2}-2x(1+x)\frac{dy}{dx}+2(1+x)y=0$. Hence, solve the differential equation: $x^2\frac{d^2y}{dx^2}-2x(1+x)\frac{dy}{dx}+2(1+x)y=x^3$ by the method of variation of parameters.

2020

1) Solve the following differential equation: $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

2) Find the orthogonal trajectories of the family of circles passing through the points $(0,2)$ and $(0,-2)$

3) Using the method of variation of parameters, solve the differential equation $y''+(1-\cot\ x)y'-y\cot\ x=\sin^2\ x$ if $y=e^{-x}$ is one solution of CF.

4) Using Laplace transform, solve the initial value problem $ty''+2ty'+2y=2$; and $y(0)=1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution?

5) Solve differential equation: $(x+1)^2y''-4(x+1)y'+6y=6(x+1)^2+\sin\ \log\ (x+1)$

6) Find the general and singular solutions of the differential equation $9p^2(2+y)^2=4(3-y)$

2019

1) Solve the differential equation $(2y\sin\ x+3y^4\sin\ x\cos\ x)dx-(4y^3\cos^2\ x+\cos\ x)dy=0$

2) Determine the complete solution of the differential $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=3x^2e^{2x}\sin\ 2x$

3) Solve the differential equation $\frac{d^2y}{dx^2}+(3\sin\ x-\cot\ x)\frac{dy}{dx}+2y\sin^2\ x=e^{-\cos\ x}\sin^2\ x$

4) Find the Laplace transforms of $t^{-1/2}$ and $t^{-1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n\in N$ is $\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$

5) Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y'' - 2xy' + 2y = x^3\sin x$ and then find the general solution of the given equation by the method of variation of parameters.

6) Obtain the singular solution of the differential equation $\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha - 2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha = 1$. Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solutions

2018

1) Solve: $y'' - y = x^2e^{2x}$

2) Solve: $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$

3) Solve

   (i) Find the Laplace transform of $f(t) = \frac{1}{\sqrt{t}}$.

   (ii) Find the Inverse Laplace transform of $\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$

4) Solve: $\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$

5) Solve: $y'' + 16y = 32\sec 2x$

6) Solve: $(1+x)^2y'' + (1+x)y' + y = 4\cos(\log(1+x))$

7) Solve the initial value problem $y'' - 5y' + 4y = e^{2t}; y(0) = \frac{19}{12}, y'(0) = \frac{8}{3}$

8) Find α and β such that $x^\alpha y^\beta$ an integrating factor of $(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation.

9) Find $f(y)$ such that $(2xe^y + 3y^2)dy + (3x^2 + f(y))dx = 0$ is exact and hence solve.

2017

1) Find the differential equation representing the entire circle in the xy-plane.

2) Solve the following simultaneous liner differential equations: $(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$.

3) If the growth rate of the population of bacteria at time t is proportional to the amount present at the time and population doubles in one week, then how much bacteria's can be expected after 4 weeks?

4) Consider the differential equation $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ where $p = \frac{dy}{dx}$ substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \frac{dv}{du}$ hence or otherwise solve the equation.

5) Solve the following initial value differential equations
$20y'' + 4y' + y = 0, y(0) = 3.2, y'(0) = 0$.

6) Solve the differential equation: $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin(x^2)$

7) Solve that following differential equation using method of variation of parameters $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2$

8) Solve the following initial value problem using Laplace transform: $\frac{d^2y}{dx^2} + 9y = r(x), y(0) = 0, y'(0) = 4$ where $r(x) = \begin{cases} 8\sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x \geq \pi \end{cases}$.

2016

1) Find a particular integral of $\frac{d^2y}{dx^2} + y = e^{x/2}\sin \frac{x\sqrt{3}}{2}$

2) Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal.

3) Solve $\{y(1 - x\tan x) + x^2\cos x\}dx - xdy = 0$

4) Using the method of variation of parameter solve the differential equation

$(D^2 + 2D + 1)y = e^{-x}\log(x)$, $\left[D \equiv \frac{d}{dx}\right]$

5) Find the general solution of the equation $x^2\frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$

6) Using Laplace transformation solve: $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$

2015

1) Solve the differential equation: $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

2) Solve the differential equation: $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

3) Find the constant a so that $(x + y)^a$ is the integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation

4) Solve (i) Obtain Laplace Inverse transform of $\left\{\ln\left(1 + \frac{1}{s^2}\right) + \frac{s}{s^2+25}e^{-5s}\right\}$

(ii) Using Laplace transform, solve $y'' + y = t, y(0) = 1, y'(0) = -2$

5) Solve the differential equation $x = py - p^2$ where $p = \frac{dy}{dx}$

6) Solve $x^4\frac{d^4y}{dx^4} + 6x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\cos(\log_e x)$

2014

1) Justify that differential equation of the form: $[y + xf(x^2 + y^2)]dx + [yf(x^2 + y^2) - x]dy = 0$ where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation $f(x^2 + y^2) = (x^2 + y^2)^2$

2) Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency.

3) Solve by the method of variation of parameters: $\frac{dy}{dx} - 5y = \sin x$

4) Solve the differential equation: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log_e x)$

5) Solve the following differential equation:
$x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x}$, when e^x is a solution to its corresponding homogeneous differential equation.

6) Find the sufficient condition for the differential equation $M(x,y)dx+N(x,y)dy=0$, to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for differential equation of $(x^2+xy)dx+(y^2+xy)dy=0$ and solve it.

7) Solve the initial value problem $\frac{d^2y}{dt^2}+y=8e^{-2t}\sin t, y(0)=0, y'(0)=0$ by using Laplace transform.

2013

1) If y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x+y)+\sin(x+y)$. Find out a relation between x and y, which is free from any derivative / differential.

2) Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n=a\sin n\theta, (r,\theta)$ being the plane polar coordinates.

3) Solve the differential equation $(5x^3+12x^2+6y^2)dx+6xydy=0$

4) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$

5) Find the general solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\ln x\sin(\ln x)$

6) By using Laplace transform method, solve the differential equation $(D^2+n^2)x=a\sin(nt+\alpha), D^2=\frac{d^2}{dt^2}$ subject to the initial conditions $x=0$ and $\frac{dx}{dt}=0$, at $t=0$, in which a, n and α are constants.

2012

1) Solve $\frac{dy}{dx}=\frac{2xye^{(x/y)^2}}{y^2\left(1+e^{(x/y)^2}\right)+2x^2e^{(x/y)^2}}$

2) Find the orthogonal trajectory of the family of curves $x^2+y^2=ax$

3) Using Laplace transforms, solve the initial value problem $y''+2y'+y=e^{-t}, y(0)=-1, y'(0)=1$

4) Show that the differential equation $(2xy\log y)dx+\left(x^2+y^2\sqrt{y^2+1}\right)dy=0$ is not exact. Find an integrating factor and hence, the solution of the equation

5) Find the general solution of the equation $y'''-y''=12x^2+6x$

6) Solve the ordinary differential equation $x(x-1)y''-(2x-1)y'+2y=x^2(2x-3)$

2011

1) Obtain the solution of the ordinary differential equation $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$

2) Determine the orthogonal trajectory of a family of curves represented by the polar equation $r = a(1 - \cos\ \theta)$, (r, θ) being the plane polar coordinates of any point.

3) Obtain Clairaut's form of the differential equation $\left(x\frac{dy}{dx} - y\right)\left(y\frac{dy}{dx} + x\right) = a^2\frac{dy}{dx}$. Also find its general solution

4) Obtain general solution of second order ordinary differential equation $y'' - 2y' + 2y = x + e^x \cos\ x$

5) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan\ 2x$

6) Use Laplace transform method to solve the following initial value problem:
$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, x(0) = 2$ and $\left.\frac{dy}{dt}\right|_{t=0} = -1$

2010

1) Consider the differential equation $y' = \alpha x, x > 0$ where α is a constant. Show that

   (i) If $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;
   (ii) If $\alpha < 0$, then every solution tends to zero as $x \to \infty$

2) Show that the differential equation $(3y^2 - x) + 2y(y^2 - 3)y' = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation

3) Verify that $\frac{1}{2}(Mx + Ny)d[\log_e (xy)] + \frac{1}{2}(Mx - Ny)d[\log_e (x/y)] = Mdx + Ndy$. Hence show

   (i) If the differential equation $Mdx + Ndy = 0$ is homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$
   (ii) If the differential equation $Mdx + Ndy = 0$ is not exact but is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$ then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx + Ny \equiv 0$;

4) Use the method of undermined coefficients to find the particular solutions of $y'' + y = \sin\ x + (1 + x^2)e^x$ and hence find its general solution.

2009

1) Find the Wronskian of the set of functions: $\{3x^3, |3x^3|\}$ on the interval $[-1,1]$ and determine whether the set is linearly dependent on $[-1,1]$

2) Find differential equation of the family of circles in xy-plane passing through $(-1,1)$ and $(1,1)$

3) Find the inverse Laplace transform of $F(s) = \ln\left(\frac{s+1}{s+s}\right)$

4) Solve: $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, y(0) = 1$

2008

1) Solve the differential equation $ydx + (x + x^3y^2)dy = 0$

2) Use method of variation of parameters to find the general solution of $x^2y'' - 4xy' + 6y = -x^4\sin\ x$

3) Using Laplace transform, solve the initial value problem
$$y'' - 3y' + 2y = 4t + e^{3t}, y(0) = 1, y'(0) = -1$$

4) Solve the differential equation $x^3y'' - 3x^2y' + xy = \sin(\ln\ x) + 1$

5) Solve the equation $y - 2xp + yp^2 = 0$, where $p = \frac{dy}{dx}$

2007

1) Solve the ordinary differential equation
$$\cos 3x\frac{dy}{dx} - 3y\sin 3x = \frac{1}{2}\sin 6x + \sin^2 3x,\ \ 0 < x < \frac{\pi}{2}$$

2) Find the solution of the equation $\frac{dy}{y} + xy^2dx = -4x\ \ dx$

3) Determine the general and singular solutions of the equation
$$y = x\frac{dy}{dx} + a\frac{dy}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$$, a being a constant.

4) Obtain the general solution of $[D^3 - 6D^2 + 12D - 8]y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right)$,

5) Solve the equation $2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$

6) Use method of variation of parameters to find general solution of equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x$

2006

1) Find the family of curves whose tangents form an angle $\frac{\pi}{4}$ with the hyperbolas
$$xy = c, c > 0$$

2) Solve the differential equation $\left(xy^2 + e^{-\frac{1}{x^3}}\right)dx - x^2ydy = 0$

3) Solve: $(1 + y^2) + \left(x - e^{-\tan^{-1}y}\right)\frac{dy}{dx} = 0$

4) Solve the equation $x^2p^2 + py(2x + y) + y^2 = 0$ using the substitution $y = u$ and $xy = v$ and find its singular solution, where $p = \frac{dy}{dx}$

5) Solve the differential equation $x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{y}{x} = 10\left(1 + \frac{1}{x^2}\right)$

6) Solve the differential equation $(D^2 - 2D + 2)y = e^x\tan x, D \equiv \frac{dy}{dx}$ by the method of variation of parameters.

2005

1) Find the orthogonal trajectory of the family of co-axial circles $x^2 + y^2 + 2gx + c = 0$, where g is the parameter.

2) Solve: $xy\frac{dy}{dx} = \sqrt{(x^2 - y^2 - x^2y^2 - 1)}$

3) Solve the differential equation: $[(x+1)^4D^3 + 2(x+1)^3D^2 - (x+1)^2D + (x+1)]y = \frac{1}{(x+1)}$

4) Solve the differential equation:
$(x^2 + y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$ where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution.

5) Solve the differential equation $(\sin x - x\cos x)y'' - x\sin x\, y' + y\sin x = 0$ given that $y = \sin x$ is a solution of this equation.

6) Solve the differential equation $x^2y'' - 2xy' + 2y = x\log x, x > 0$ by variation of parameters

2004

1) Find the solution of the following differential equation $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$

2) Solve: $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

3) Solve: $(D^4 - 4D^2 - 5)y = e^x(x + \cos x)$

4) Reduce the equation $(px - y)(py + x) = 2p$, to Clairaut's equation and solve it.

5) Solve: $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$

6) Solve the following differential equation: $(1 - x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1 + x^2)y = x$

2003

1) Show that the orthogonal trajectory of a system of confocal ellipses is self-orthogonal

2) Solve: $x\frac{dy}{dx} + y\log y = xye^x$

3) Solve $(D^5 - D)y = 4(e^x + \cos x + x^3)$, where $D \equiv \frac{dy}{dx}$

4) Solve the differential equation $(px^2 + y^2)(px + y) = (p+1)^2$, where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions

5) Solve $(1 + x^2)y'' + (1 + x)y' + y = \sin 2[\log(1 + x)]$

6) Solve the differential equation $x^2y'' - 4xy' + 6y = x^4\sec^2 x$ by variation of parameters.

2002

1) Solve: $x\frac{dy}{dx} + 3y = x^3y^2$

2) Find the values of λ for which all solutions of $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - \lambda y = 0$ tend to zero as $x \to \infty$.

3) Find the value of constant λ such that the following differential equation becomes exact. $(2xe^y + 3y^2)\frac{dy}{dx} + (3x^2 + \lambda e^y) = 0$. Further, for this value of λ, solve the equation.

4) Solve: $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}$

5) Using the method of variation of parameters, find the solutions of $\frac{d^2y}{d^2x} - 2\frac{dy}{dx} + y = xe^x \sin x$ with $y(0) = 0$ and $\left(\frac{dy}{dx}\right)_{x=0} = 0$

6) Solve: $(D-1)(D^2 - 2D + 2)y = e^x$ where $D \equiv \frac{dy}{dx}$

2001

1) A continuous function $y(t)$ satisfies the differential equation

$$\frac{dy}{dx} = \begin{cases} 1 + e^{1-t}, & 0 \le t < 1 \\ 2 + 2t - 3t^2, & 1 \le t < 5 \end{cases}$$

If $y(0) = -e$, find $y(2)$

2) Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log_e x$

3) Solve: $\frac{dy}{dx} + \frac{y}{x}\log_e y = \frac{y(\log_e y)^2}{x^2}$

4) Find the general solution of $ayp^2 + (2x - b)p - y = 0, a > 0$

5) Solve: $(D^2 + 1)^2 y = 24x\cos x$ given that $y = Dy = D^2y = 0$ and $D^3y = 12$ when $x = 0$

6) Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$