SuccessClap
Best Coaching for UPSC Mathematics

UPSC CSE Mathematics: Previous Year Questions: Vector Analysis

2025

1) If $u = x + y + z, v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, then show that grad u, grad v and grad w are coplanar.
2) Find the absolute value of the directional derivative of $\phi(x, y, z) = x^2y^2z^2$ at the point $(1,1,-1)$ in the direction of the tangent to the curve $\mathrm{x} = e^{\mathrm{t}}, \mathrm{y} = 2\sin\ \mathrm{t} + 1, \mathrm{z} = \mathrm{t} - \cos\ \mathrm{t}$, at $\mathrm{t} = 0$.
3) If $\nabla \cdot \vec{E} = 0, \nabla \cdot \vec{H} = 0, \nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t}$ and $\nabla \times \vec{H} = \frac{\partial \vec{E}}{\partial t}$, then show that $\nabla^2 \vec{H} = \frac{\partial^2 \vec{H}}{\partial t^2}$ and $\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2}$.
4) Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2dy]$, where C is the boundary of the region bounded by the curves $\mathrm{y} = \mathrm{x}$ and $\mathrm{y} = \mathrm{x}^2$.
5) Verify Gauss's divergence theorem for $\vec{F} = \left[(x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}\right]$, taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

2024

1) At any time t (in seconds), the coterminous edges of a variable parallelepiped are represented by the vectors

$$\bar{\alpha} = t\hat{\imath} + (t+1)\hat{\jmath} + (2t+1)\hat{k}$$
$$\bar{\beta} = 2t\hat{\imath} + (3t-1)\hat{\jmath} + t\hat{k}$$
$$\bar{\gamma} = \hat{\imath} + 3t\hat{\jmath} + \hat{k}$$

What is the rate of change of the vectorial area of the parallelogram, whose coterminous edges are $\bar{\alpha}$ and $\bar{\gamma}$? Also find the rate of change of the volume of the parallelepiped at $t = 1$ second.

2) Let C be a plane curve $\bar{r}(t) = f(t)\hat{\imath} + g(t)\hat{\jmath}$, where f and g have second-order derivatives. Show that the curvature at a point is given by

$$K = \frac{|f'(t)g''(t) - g'(t)f''(t)|}{([f'(t)]^2 + [g'(t)]^2)^{3/2}}$$

What is the value of torsion τ at any point of this curve?

3) Show that the principal normals at two consecutive points of a curve do not intersect unless torsion τ is zero.

4) State Stokes' theorem and verify it for the vector field $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + zx\hat{k}$ over the surface S, which is the upwardly oriented part of the cylinder $z = 1 - x^2$, for $0 \leq x \leq 1, -2 \leq y \leq 2$.

5) Using Gauss divergence theorem, evaluate the integral

$$\iint_S \left(y^2\hat{\imath} + xz^3\hat{\jmath} + (z-1)^2\hat{k}\right) \cdot \hat{n}dS$$

over the region bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 1$ and $z = 5$.

2023

1) If $\vec{a} = \sin\theta\hat{\imath} + \cos\theta\hat{\jmath} + \theta\hat{k}$ $\vec{b} = \cos\theta\hat{\imath} - \sin\theta\hat{\jmath} - 3\hat{k}$ $\vec{c} = 2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$ then find the values of the derivative of the vector function $\vec{a} \times (\vec{b} \times \vec{c})$ w.r.t. θ at $\theta = \frac{\pi}{2}$ and $\theta = \pi$.

2) Evaluate the integral $\iint_S \left(3y^2z^2\hat{\imath} + 4z^2x^2\hat{\jmath} + z^2y^2\hat{k}\right) \cdot \hat{n}dS$, where S is the upper part of the surface $4x^2 + 4y^2 + 4z^2 = 1$ above the plane $z = 0$ and bounded by the xy-plane. Hence, verify Gauss-Divergence theorem.

3) For a scalar point function ϕ and vector point function $\vec{f}$, prove the identity $\nabla \cdot (\phi\vec{f}) = \nabla\phi \cdot \vec{f} + \phi(\nabla \cdot \vec{f})$. Also find the value of $\nabla \cdot \left(\frac{f(r)}{r}\vec{r}\right)$ and then verify stated identity

2022

1) Show that $\vec{A} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Also find ϕ such that $\vec{A} = \nabla\phi$.

2) Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary curve of the region defined by $x = 0, y = 0$, where C is the boundary curve of the region defined by $x = 0, y = 0, x + y = 1$

3) Verify Stokes' theorem for $\vec{F} = x\hat{\imath} + z^2\hat{\jmath} + y^2\hat{k}$ over the plane surface: $x + y + z = 1$ lying in the first octant.

4) Using Gauss' divergence theorem, evaluate $\iint_S \vec{F} \cdot \vec{n}dS$, where $\vec{F} = x\hat{\imath} - y\hat{\jmath} + (z^2 - 1)\hat{k}$ and S is the cylinder formed by the surfaces $z = 0, z = 1, x^2 + y^2 = 4$

2021

1) Show that $\nabla^2\left[\nabla \cdot \left(\frac{\vec{r}}{r}\right)\right] = \frac{2}{r^4}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

2) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is an arbitrary closed curve in the xy-plane and $\vec{F} = \frac{-y\hat{\imath} + x\hat{\jmath}}{x^2+y^2}$

3) Verify Gauss divergence theorem for $\vec{F} = 2x^2y\hat{\imath} - y^2\hat{\jmath} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$.

4) Using Stokes' theorem, evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$, where $\vec{F} = (x^2 + y - 4)\hat{\imath} + 3xy\hat{\jmath} + (2xy + z^2)\hat{k}$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy-plane. Here, $\hat{n}$ is the unit outward normal vector on S.

2020

1) For what value of a, b, c is the vector field $\bar{V} = (-4x - 3y + az)i + (bx + 3y + 5z)j + (4x + cy + 3z)k$ irrotational? Hence, express $\bar{V}$ as the gradient of a scalar function ϕ determine ϕ

2) For the vector function $\bar{A}$ where $\bar{A} = (3x^2 + 6y)i - 14yzj + 20xz^2k$, calculate $\int_C \bar{A} \cdot d\bar{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following paths:
   (i) $x = t, y = t^2, z = t^3$
   (ii) Straight lines joining $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$
   (iii) Straight line joining $(0,0,0)$ to $(1,1,1)$ is the result same in all the cases? Explain the reason.

3) Verify the stokes theorem for the vector field $\bar{F} = xyi + yzj + xzk$ on the surface S which is the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1, -2 \le y \le 2$; S is oriented upwards.

4) Evaluate the surface integral $\iint_s \nabla \times \bar{F}.nds$ for $\bar{F} = yi + (x - 2xz)j - xyk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane

2019

1) Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t^2, y = t^2, z = t^3$ at the point $(1,1,1)$

2) Find the circulation of $\vec{F}$ round the curve C where $\vec{F} = (2x + y^2)i(3y - 4x)j$ and C is curve $y^2 = x$ from $(0,0)$ to $(1,1)$ and curve $y = x^2$ from $(1,1)$ to (0,0)

3) Find the radius of curvature and radius of torsion of the helix $x = a\cos u, y = a\sin u, z = au\tan \alpha$

4) State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4xi - 2y^2j + z^2k$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$

5) Evaluation by Stoke's theorem $\oint_C e^x dx + 2ydy - dz$ where C is the curve $x^2 + y^2 = 4, z = 2$.

2018

1) Find the angle between the tangent at a general point of the curve whose equations are $x = 3t, y = 3t^2, z = 3t^3$ and the line $y = z - x = 0$

2) Let $\vec{v} = v_1\vec{\imath} + v_2\vec{\jmath} + v_3\vec{k}$.Show that curl $(\text{cur l } \vec{v}) = \text{grad } (\text{div } \vec{v}) - \nabla^2\vec{v}$.

3) Evaluate the line integral $\int_C -y^3dx + x^3dy + z^3dz$ using stokes theorem. Here C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. The orientation on C corresponds to counterclockwise motion in the xy-plane.

4) Let $\vec{F} = xy^2\vec{\imath} + (y + x)\vec{\jmath}$ Integrate $(\nabla \times \vec{F}) \cdot \vec{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and $y = x$ using Green's theorem.

5) Find the curvature and torsion of the curve $\vec{r} = a(u - \sin u)\vec{\imath} + a(1 - \cos u)\vec{\jmath} + bu\vec{k}$

6) If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate

$\iint_s [(x + z)dydz + (y + z)dzdx + (x + y)dxdy]$ using gauss' divergence theorem.

2017

1) For what values of the constant a, b and c the vector $\bar{V} = (x + y + az)\hat{\imath} + (bx + 2y - z)j + (-x + cy + 2z)k$ is irrotational. Find the divergence in cylindrical coordinates of the vector with these values.

2) The position vector of a moving point at time t is $\bar{r} = \sin t\hat{\imath} + \cos 2tj + (t^2 + 2t)k$. Find the components of acceleration $\bar{a}$ in the direction parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time $t = 0$.

3) Find the curvature vector and its magnitude at any point $\bar{r} = (\theta)$ of the curve $\bar{r} = (a\cos \theta, a\sin \theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 - z^2 = a^2$.

4) Evaluate the integral $\iint_S \vec{F}.nds$ where $\vec{F} = 3xy^2\hat{\imath} + (yx^2 - y^3)j + 3zx^2K$ and S is a surface of the cylinder $y^2 + z^2 \leq 4, -3 \leq x \leq 3$ using divergence theorem.

5) Using Green theorem evaluate the $\int_C F(\vec{r}) \cdot d\vec{r}$ counterclockwise where $F(\vec{r}) = (x^2 + y^2)\hat{\imath} + (x^2 - y^2)j$ and $d\vec{r} = \mathrm{d}x\hat{\imath} + dyj$ and the curve C is the boundary off the region $R = \{(x, y) \mid 1 \leq y \leq 2 - x^2\}$.

2016

1) Prove that the vector $\vec{a} = 3\vec{\imath} + \hat{\jmath} - 2\hat{k}, \vec{b} = -\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, \vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ can from the sides of a triangle find the length of the medians of the triangle

2) Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$

3) Prove that $\oint_c f d\vec{r} = \iint_S d\vec{S} \times \nabla f$

4) For the of cardioid $r = a(1 + \cos \theta)$ show that the square of the radius of curvature at any point (r, θ) is proportion to r. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$.

2015

1) Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$

2) A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$. Verify that the field is irrotational or not. Find the scalar potential.

3) Evaluate $\int_C e^{-x}(\sin\ ydx + \cos\ ydy)$, where C is the rectangle with vertices $(0,0)(\pi,0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$

4) Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at $(1, -1, 2)$.

2014

1) Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos\ t\hat{\imath} + t\sin\ t\hat{\jmath}, 0 \le t \le 2\pi$. Give its magnitude also.

2) Evaluate by Stokes' theorem $\int_\Gamma (ydx + zdy + xdz)$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0, x + y = 2a$ starting from $(2a, 0, 0)$ and then going below the z-plane.

2013

1) Show the curve $\vec{x}(t) = t\hat{\imath} + \left(\frac{1+t}{t}\right)\hat{\jmath} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane.

2) Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x, y, z) from the origin, n being a constant and ∇^2 being the Laplace operator

3) A curve in space is defined by the vector equation $\vec{r} = t^2\hat{\imath} + 2t\hat{\jmath} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$

4) By using Divergence Theorem of Gauss, evaluate the surface integral $\iint (a^2x^2 + b^2y^2 + c^2z^2)^{-\frac{1}{2}}dS$, where S is the surface e of the ellipsoid $ax^2 + by^2 + cz^2 = 1, a, b$ and c being all positive constants.

5) Use Stokes' theorem to evaluate the line integral $\int_C (-y^3dx + x^3dy - z^3dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$

2012

1) If $\vec{A} = x^2yz\vec{\imath} - 2xz^3\vec{\jmath} + xz^2\vec{k}, \vec{B} = 2z\vec{\imath} + y\vec{\jmath} - x^2\vec{k}$ find the value of $\frac{\partial^2}{\partial x\,\partial y}(\vec{A}\ \mathrm{X}\ \vec{B})$ at $(1, 0, -2)$

2) Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$. Show that the curvature and torsion are equal for this curve.

3) Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$

4) If $\vec{F} = y\vec{\imath} + (x - 2xz)\vec{\jmath} - xy\vec{k}$, evaluate $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}d\vec{s}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane.

2011

1) For two vectors $\vec{a}$ and $\vec{b}$ give respectively by $\vec{a} = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$ and $\vec{b} = \sin\ 5t\hat{\imath} - \cos\ t\hat{\jmath}$ determine:(i) $\frac{d}{dt}(\vec{a}\cdot\vec{b})$ and (ii) $\frac{d}{dt}(\vec{a}\times\vec{b})$

2) If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = \text{grad}\ v$, find the value of $\vec{f}$. curl $\vec{f}$

3) Examine whether the vectors $\nabla u, \nabla u$ and ∇w are coplanar, where u, v and w are the scalar functions defined by: $u = x + y + z$ $\quad v = x^2 + y^2 + z^2$ and $w = yz + zx + xy$

4) If $\vec{u} = 4y\hat{\imath} + x\hat{\jmath} + 2z\hat{k}$ calculate the double integral $\iint (\nabla \times \vec{u}) \,.\, d\vec{s}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \geq 0$

5) If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector. $r^n\vec{r}$ is

   (i) irrotational, (ii) solenoidal

6) Verify Gauss' Divergence Theorem for the vector $\vec{v} = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$ taken over the

   cube $0 \leq x, y, z \leq 1$

2010

1) Find the directional derivative of $f(x, y) = x^2y^3 + xy$ at the point $(2,1)$ in the direction of a unit vector which makes an angle or $\frac{\pi}{3}$ with the x-axis.

2) Show that the vector field defined by the vector function $\vec{v} = xyz(yz\vec{\imath} + xy\vec{\jmath} + xy\vec{k})$ is conservative.

3) Prove that $\text{div}(f\vec{V}) = f(\text{div}\,\vec{V}) + (\text{grad} f).\vec{V}$ where f is a scalar function.

4) Use the divergence theorem to evaluate $\iint_S \vec{V}.\vec{n}dA$ where $\vec{V} = x^2z\,\vec{\imath} + y\vec{\jmath} - xz^2\vec{k}$ and S is he boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$.

5) Verify Green's theorem for $e^{-x}\sin\ ydx + e^{-x}\cos\ y$ by the path of integration being the boundary of the square whose vertices are $(0,0), \left(\frac{\pi}{2}\ ,\ 0\right)\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\left(0,\ \frac{\pi}{2}\right)$

6) Find κ/τ for the curve $\vec{r}(t) = a\cos\ t\vec{\imath} + a\sin\ t\vec{\jmath} + bt\vec{k}$

2009

1) Show that $\text{div}\ (\text{grad} r^n) = n(n+1)r^{n-2}$ where $r = \sqrt{x^2 + y^2 + z^2}$.

2) Find the directional derivative of

   (i) $4xz^3 - 3x^2y^2z^2$ at $(2, -1,2)$ along z-axis

   (ii) $x^2yz + 4xz^2$ at $(1, -2,1)$ in the direction of $2\hat{\imath} - \hat{\jmath} - 2\hat{\kappa}$.

3) Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$ under the field of force of given by $\bar{F} = (2x - y + z)\hat{\imath} + (x + y - z^2)\hat{\jmath} + (3x - 2y + 4z)\hat{k}$.

4) Using divergence theorem, evaluate $\iint_S \bar{A} \cdot d\bar{S}$ where $\bar{A} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$

5) Find the value of $\iint_s (\vec{\nabla} \times \vec{f}) \cdot d\vec{s}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane $z = 0$, when $\vec{F} = (y^2 + z^2 - x^2)\hat{\imath} + (z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - z^2)\hat{k}$

2008

1) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$.

2) Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from $(1-2,1)$ to $(3,1,4)$.

3) Prove that $\nabla^2 f(x) = \frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$. Hence find $f(x)$ such that $\nabla^2 f(r) = 0$.

4) Show that for the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$ the curvature and torsion are same at every point.

5) Evaluate $\int_C \vec{A} \cdot d\vec{r}$ along the curve $x^2 + y^2 = 1$ $z = 1$ from $(0,1,1)$ to $(1,0,1)$ if $\vec{A} = (yz + 2x)\hat{\imath} + xz\hat{\jmath} + (xy + 2z)\hat{k}$

6) Evaluate $\iint_S \vec{F} \cdot \vec{n} dS$ where $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and S is the surface of the cylinder bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$.

2007

1) If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}$, $r = |\vec{r}|$, determine grad (r^{-1}) in terms of $\hat{r}$ and r.

2) Find the curvature and torsion at any point of t curve $x = a\cos 2t, y = a\sin 2t, z = 2a\sin t$.

3) For any constant vector, show that the vector $\vec{a}$ represented by curl $(\vec{a} \times \vec{r})$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z) measured from the origin.

4) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + x\hat{k}$ find the value(s) of n in order that $r^n\vec{r}$ may be (i) solenoidal (ii) irrotational

5) Determine $\int_c (ydx + zdy + xdz)$ by using Stoke's theorem, where C is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2, x + y = 2a$ that starts from the point $(2a, 0,0)$ goes at first below the z-plane.

2006

1) Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x^3$ at the point $(1,2,-1)$ has maximum magnitude 64 in the direction parallel to z-axis.

2) If $\bar{A} = 2\bar{\imath} + \bar{k}$, $\bar{B} = \bar{\imath} + \bar{\jmath} + \bar{k}$, $\bar{C} = 4\bar{\imath} - 3\bar{\jmath} - 7\bar{k}$ determine a vector $\bar{R}$ satisfying the vector equation $\bar{R} \times \bar{B} = \bar{C} \times \bar{B}$ and $\bar{R} \cdot \bar{A} = 0$

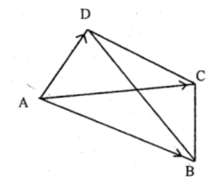
3) Prove that $r^n \bar{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n + 3 = 0$

4) If the unit tangent vector $\bar{t}$ and binormal $\bar{b}$ make angles θ and ϕ respectively with a constant unit vector $\bar{a}$, prove that $\frac{\sin\theta}{\sin\phi} \cdot \frac{d\theta}{d\phi} = -\frac{k}{\tau}$.

5) Verify Stokes' theorem for the function $\bar{F} = x^2\,\hat{\imath} - xy\,\hat{\jmath}$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a$ and $y = a$, $a > 0$.

2005

1) Show that the volume of the tetrahedron $ABCD$ is $\frac{1}{6}(\overrightarrow{AB} \times \overrightarrow{AC}).\overrightarrow{AD}$. Hence , find the volume of the tetrahedron with vertices $(2,2,2)$, $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$



2) Prove that the curl of a vector field is independent of the choice of coordinates

3) The parametric equation of a circular helix is $r = a\cos u\hat{\imath} + a\sin u\hat{\jmath} + cu\hat{k}$ where c is a constant and u is a parameter. Find the unit tangent vector $\hat{t}$ at the point u and the arc length measured form $u = 0$. Also find $\frac{d\hat{t}}{ds}$, where s is the arc length.

4) Show that curl $\left(k \times \text{grad}\ \frac{1}{r}\right) + \text{grad}\ \left(k \cdot \text{grad}\ \frac{1}{r}\right) = 0$ where r is the distance from the origin and k is the unit vector in the direction OZ

5) Find the curvature and the torsion of the space curve

$$x = a(3u - u^3)$$
$$y = 3au^2$$
$$z = a(3u + u^3)$$

6) Evaluate $\oiint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$ by Gauss divergence theorem, where S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by $z = 0$ and $z = b$.

2004

1) Show that if $\bar{A}$ and $\bar{B}$ are irrotational, then $\bar{A} \times \bar{B}$ is solenoidal.

2) Show that the Frenet-Serret formulae can be written in the form

$\frac{d\bar{T}}{ds} = \bar{\omega} \times \bar{T}, \quad \frac{d\bar{N}}{ds} = \bar{\omega} \times \bar{N} \quad \frac{d\bar{B}}{dx} = \bar{\omega} \times \bar{B}$, where $\bar{\omega} = \tau\bar{T} + k\bar{B}$.

3) Prove the identity $\nabla(\bar{A} \cdot \bar{B}) = (\bar{B} \cdot \nabla)\bar{A} + (\bar{A} \cdot \nabla)\bar{B} + \bar{B} \times (\nabla \times \bar{A}) + \bar{A} \times (\nabla \times \bar{B})$

4) Derive the identity $\iiint_v (\phi\nabla^2\psi - \psi\nabla^2\phi)dV = \iint_s (\phi\nabla\psi - \psi\nabla\phi).\hat{n}dS$ where V is the volume bounded by the closed surface S.

5) Verify Stokes' theorem for $\hat{f} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

2003

1) Show that if a', b' and c' are the reciprocals of the non-coplanar vectors a, b and c, then any vector r may be expressed as $r = (r.a')a + (r.b')b + (r.c')c$.

2) Prove that the divergence of a vector field is invariant w.r. to co-ordinate transformations.

3) Let the position vector of a particle moving on a plane curve be $r(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions.

4) Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A \times (\nabla \times A)$ where $\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

5) Find the radii of curvature and torsion at a point of intersection of the surface

$$x^2 - y^2 = c^2, y = x\tanh\left(\frac{z}{c}\right).$$

6) Evaluate $\iint_s$ curtA. ds where S is the open surface $x^2 + y^2 - 4x + 4z = 0, z \geq 0$ and $A = (y^2 + z^2 - x^2)\hat{\imath} + (2z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - 3z^2)\hat{k}$

2002

1) Let $\bar{R}$ be the unit vector along the vector $\bar{r}(t)$

Show that $\bar{R} \times \frac{\overline{dR}}{dt} = \frac{\bar{r}}{r^2} \times \frac{\overline{dr}}{dt}$ where $r = |\bar{r}|$

2) Find the curvature k for the space curve $x = a\cos\,\theta, y = a\sin\,\theta, z = a\theta\tan\,\alpha$

3) Show that curl (culr $\bar{v}$) $=$ grad (div $\bar{v}$) $- \nabla^2\bar{v}$.

4) Let D be a closed and bounded region having boundary S. Further, let f is a scalar function having second partial derivatives defined on it.

Show that $\iint_s$(f grad f)$.\hat{n}ds = \iiint_v (\,|\,\text{gradf}\,|^2 + f\nabla^2 f\,)\ dv$ .Hence or otherwise evaluate $\iint_s$(f grad f)$.\hat{n}ds$ for $f = 2x + y + 2z$ over $s = x^2 + y^2 + z^2 = 4$

5) Find the values of constants a, b and c such that the maximum value of directional derivative of $f = axy^2 + byz + cx^2z^2$ at $(1, -1, 1)$ is in the direction parallel to y-axis and has magnitude 6

2001

1) Find the length of the arc of the twisted curve $r = (3t, 3t^2, 2t^3)$ from the point $t = 0$ to the point $t = 1$ Find also the unit tangent t, unit normal n and the unit binormal b at $t = 1$.

2) Show that $\mathrm{curl}\ \frac{a\times r}{r^3} = -\frac{a}{r^3} + \frac{3r}{r^5}(a.r)$ where a is constant vector.

3) Find the directional derivative of $f = x^2yz^3$ along $x = e^{-t}\ , y\ = 1 + 2\sin\ t, z = t - \cos\ t$ at $t = 0$

4) Show that the vector field defined by $F = 2xyz^3i + x^2z^3j + 3x^2yz^2k$ is irrotational. Find also the scalar u such that $F = \mathrm{grad}\ u$

5) Verify Gauss' divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2 + y^2 = 4,\ \ z = 0$ and $z = 3$.