SuccessClap

Best Coaching for UPSC Mathematics

Note: Solutions of all the questions are available in sequence in Crash Course. There is very less time for Mains exam and it is not possible to solve all problems, hence it is advisable to go through the solutions (Quick glance) of all these important questions at once. All topics, Important questions and important PYQ questions are covered.

Important Questions for UPSC Maths 2026

Partial Differential Equations

Formation of PDE

1) Find the differential equation of the set of all right circular cones whose axes coincide with z-axis.
2) Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.
3) Find the partial differential equation of all planes which are at a constant distance ' a 'from the origin.
4) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$. What is the order of this partial differential equation?
5) Find the differential equation of all surfaces of revolution having z-axis as the axis of rotation.
6) Equation of any cone with vertex at $P(a, b, c)$ is of the form $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. Find the differential equation of the cone.

7) Solve $(x^2 + 2y^2)p - xyq = xz$
8) Solve $z(z^2 + xy)(px - qy) = x^4$
9) Solve $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$
10) Solve $\{(b - c)/a\}yzp + \{(c - a)/b\}zxq = \{(a - b)/c\}xy$.
11) Solve $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^2 - y^3)$
12) Solve $(2x^2 + y^2 + z^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = x^2 + y^2 + 2z^2 - yz - zx - 2xy$.
13) Solve $\{my(x + y) - nz^2\}(\partial z/ \partial x) - \{lx(x + y) - nz^2\}(\partial z/ \partial y) = (lx - my)z$

14) Solve $px(z-2y^2) = (z-qy)(z-y^2-2x^2)$.

15) Solve $(x+y-z)(p-q) + a(px-qy+x-y) = 0$.

16) Find the surface whose tangent planes cut off an intercept of constant length k from the axis of z.

17) Find the integral surface of the linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0, z=1$.

18) Find the equation of the integral surface of the differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$, which pass through the circle $z=0, x^2+y^2=2x$

19) Find the integral surface of the partial differential equation $(x-y)p + (y-x-z)q = z$ through the circle $z=1, x^2+y^2=1$.

20) Find the equation of the integral surface of the differential equation $(x^2-yz)p + (y^2-zx)q = z^2-xy$ which passes through the line $x=1, y=0$.

21) Find the equation of surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2+z^2=1, x+z=2$.

22) Find the general integral of the partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x=1, y=0$.

23) Find the integral surface of $x^2p + y^2q + z^2 = 0, p = \partial z/\partial x, q = \partial z/\partial y$ which passes through the hyperbola $xy = x+y, z=1$.

24) Find the integral surface of the linear first order partial differential equation $yp + xq = z-1$ which passes through the curve $z = x^2+y^2+1, y=2x$

25) Find the integral surface of the partial differential equation $(x-y)y^2p + (y-x)x^2q = (x^2+y^2)z$ passing through the curve $xz = a^3, y=0$.

26) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1, z=1$.

27) Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2+y^2+z^2) = C_1y^2$.

28) Find the surface which is orthogonal to the one parameter system $z = cxy(x^2+y^2)$ which passes through the hyperbola $x^2-y^2=a^2, z=0$

29) Find the family orthogonal to $\phi[z(x+y)^2, x^2-y^2] = 0$.

30) Find the family of surfaces orthogonal to the family of surfaces given by the differential equation $(y+z)p + (z+x)q = x+y$.

31) Solve $x_2x_3p_1 + x_3x_1p_2 + x_1x_2p_3 + x_1x_2x_3 = 0$

32) Prove that if $x_1^3 + x_2^3 + x_3^3 = 1$ when $z = 0$, the solution of the equation $(s - x_1)p_1 + (s - x_2)p_2 + (s - x_2)p_2 + (s - x_3)p_3 = s - z$ can be given in the form $s^3\{(x_1 - z)^3 + (x_2 - z)^3 + (x_3 - z)^3\}^4 = (x_1 + x_2 + x_3 - 3z)^3$, where $s = x_1 + x_2 + x_3 + z$ and $p_i = \partial z/\partial x_i, i = 1,2,3$.

33) Show that the equations $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.

34) Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.

35) Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ which is homogeneous in x, y, z.

36) Find a complete integral of $z^2(p^2z^2 + q^2) = 1$

37) Find the complete integrals of following equations: $q = (z + px)^2$

38) Find a complete integral of $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$.

39) Find the complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.

40) Find a complete integral $p^2 + q^2 - 2px - 2qy + 1 = 0$.

41) Find a complete integral of $p^2 + q^2 - 2px - 2qy + 2xy = 0$.

42) Find a complete integral of $p^2x + q^2y = z$.

43) Find complete integral of the equation $q = \{(1 + p^2)/(1 + y^2)\}x + yp(z - px)^2$.

44) Solve $z = (1/2) \times (p^2 + q^2) + (p - x)(q - y)$

45) Solve by Charpit's $p^2x(x - 1) + 2pqxy + q^2y(y - 1) - 2pxz - 2qyz + z^2 = 0$.

46) Find the complete integral of the following partial differential equations $px^5 - 4q^2x^2 + 6x^2z - 2 = 0$.

47) Find the complete integral of $z^2p^2y + 6zpxy + 2zqx^2 + 4x^2y = 0$.

48) Find the complete integral of $(1 - x^2)yp^2 + x^2q = 0$.

49) Find the complete integral of $(y - x)(qy - px) = (p - q)^2$.

50) Find the complete integral of $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$.

51) Find the complete integral of $p^3\sec^6 x + z^2q^2\operatorname{cosec}^4 y = z^6$

52) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y} + \cos(x + 2y)$.

53) Solve $(D^2 + 3DD' + 2D'^2)z = x + y$, by expanding the particular integral in ascending powers of D as well as in ascending powers of D'.

54) Solve $(D^3 - 7DD'^2 - 6D'^3)z = x^2 + xy^2 + y^3 + \cos(x - y)$

55) Solve $\partial^3u/\partial x^3 + \partial^3u/\partial y^3 + \partial^3u/\partial z^3 - 3(\partial^3u/\partial x\,\partial y\,\partial z) = x^3 + y^3 + z^3 - 3xyz$.

56) Solve $(a)(D^2 - DD' - 2D'^2)z = (y - 1)e^x$.

57) Solve $(D^2 - 4D'^2)z = (4x/y^2) - (y/x^2)$

58) Solve $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$.

59) Solve $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$.

60) Solve $r - t = \tan^3 x\tan y - \tan x\tan^3 y$ or $(D^2 - D'^2)z = \tan^3 x\tan y - \tan x\tan^3 y$

61) Solve $(D^2 + DD' - 6D'^2)z = x^2\sin(x + y)$.

62) Find a surface passing through the two lines $z = x = 0, z - 1 = x - y = 0$ satisfying $r - 4s + 4t = 0$. (UPSC 2023)

63) Find the surface satisfying the equation $r + t - 2s = 0$ and the conditions that $bz = y^2$ when $x = 0$ and $az = x^2$ when $y = 0$.

64) Find a surface satisfying $r - 2s + t = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$.

65) A surface is drawn satisfying $r + t = 0$ and touching $x^2 + z^2 = 1$ along its section by $y = 0$. Obtain its equation in the form $x^2(x^2 + z^2 - 1) = y^2(x^2 + z^2)$.

66) Solve $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$.

67) Solve $D(D + D' - 1)(D + 3D' - 2)z = x^2 - 4xy + 2y^2$.

68) Solve $r - 3s + 2t - p + 2q = (2 + 4x)e^{-y}$

69) Solve $(D^2 - DD' + D' - 1)z = 1 + xy + e^y + \cos(x + 2y)$

70) Find a surface satisfying $r + s = 0$, i.e., $(D^2 + DD')z = 0$ and touching the elliptic paraboloid $z = 4x^2 + y^2$ along its section by the plane $y = 2x + 1$.

71) Find the general solution of $x^2(\partial^2 z/\partial x^2) + 2xy(\partial^2 z/\partial x\,\partial y) + y^2(\partial^2 z/\partial y^2) + nz = n\{x(\partial z/\partial x) + y(\partial z/\partial y)\} + x^2 + y^2 + x^3$.

72) Solve $x^2(\partial^2 z/\partial x^2) + 2xy(\partial^2 z/\partial x\,\partial y) + y^2(\partial^2 z/\partial y^2) = (x^2 + y^2)^{n/2}$.

73) Solve $(x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD')z = \log(y/x) - (1/2)$.

74) $(x^2D^2 - 2xyDD' - 3y^2D'^2 + xD - 3yD')z = x^2y\cos(\log x^2)$

75) Solve $\frac{1}{x^2}\frac{\partial^2 z}{\partial x^2} - \frac{1}{x^3}\frac{\partial z}{\partial x} = \frac{1}{y^2}\frac{\partial^2 z}{\partial y^2} - \frac{1}{y^3}\frac{\partial z}{\partial y}$.

Surface Problems

76) Find a surface satisfying equation $2x^2r - 5xys + 2y^2t + 2(px + qy) = 0$ and touching the hyperbolic paraboloid $z = x^2 - y^2$ along its section by the plane $y = 1$.

77) Find the surface satisfying $t = 6x^2y$ containing two lines $y = 0 = z$ and $y = 2 = z$.

78) Find the surface passing through the parabolas $z = 0, y^2 = 4ax$ and $z = 1, y^2 = -4ax$ and satisfying the equation $xr + 2p = 0$.

79) Show that a surface satisfying $r = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$ is $z = x^3 + y^3 + (x + y + 1)^2$.

80) Show that a surface passing through the circle $z = 0, x^2 + y^2 = 1$ and satisfying the differential equation $s = 8xy$ is $z = (x^2 + y^2)^2 - 1$.

81) Show that a surface of revolution satisfying the differential equation $r = 12x^2 + 4y^2$ and touching the plane $z = 0$ is $z = (x^2 + y^2)^2$.

Canonical

82) Reduce $\partial^2 z/ \partial x^2 = x^2(\partial^2 z/ \partial y^2)$ to canonical form.

83) Reduce $x^2(\partial^2 z/ \partial x^2) - y^2(\partial^2 z/ \partial y^2) = 0$ to canonical form and hence solve it.

84) Reduce the equation $\partial^2 z/ \partial x^2 + 2(\partial^2 z/ \partial x\, \partial y) + \partial^2 z/ \partial y^2 = 0$ to canonical form and solve it.

85) Reduce the equation $y^2(\partial^2 z/ \partial x^2) - 2xy(\partial^2 z/ \partial x\, \partial y) + x^2(\partial^2 z/ \partial x^2) = (y^2/x)(\partial z/ \partial x) + (x^2/y)(\partial/ \partial y)$ to canonical form and hence solve it.

86) Reduce the equation $x^2 r - 2xys + y^2 t - xp + 3yq = 8y/x$ to canonical form.

87) Reduce to canonical forms: $\partial^2 z/ \partial x^2 + x^2(\partial^2 z/ \partial y^2) = 0$ or $r + x^2 t = 0$

88) Reduce $y^2(\partial^2 z/ \partial x^2) + x^2(\partial^2 z/ \partial y^2) = 0$ to canonical form

89) Reduce $x(\partial^2 z/ \partial x^2) + \partial^2 z/ \partial y^2 = x^2$

90) Reduce $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\, \partial y} + 5\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} - 3z = 0$ to canonical form.

String Eqns

91) A slightly stretched string with fixed ends $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement y at any distance x from one end and at any time t.

92) The points of trisection of a tightly stretched string of length l with fixed ends are pulled aside through a distance d on opposite sides of the position of equilibrium and the string is released from rest. Obtain the displacement of the string at any subsequent time and show that the midpoint of the string always remains at rest.

93) A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial velocities $v = \begin{cases} \frac{cx}{l} & \text{in } 0 < x < l \\ \frac{c}{l}(2l - x) & \text{in } l < x < 2l \end{cases}$ x being the distance from an end point. Find the displacement of the string.

Heat Eqns

94) A uniform rod of length l through which heat flows is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given by $k\sin^3 \frac{\pi x}{l}, 0 < x < l$, find the temperature distribution in the bar at any time t.

95) If both the ends of a bar of length a are at temperature zero and the initial temperature is to be prescribed function $f(x)$ in the bar, then find the temperature at a subsequent time t.

96) A bar 100 cm long, with insulated sides, has its ends kept at $0°\mathrm{C}$ and $100°\mathrm{C}$ until steady state conditions prevail. The two ends are the suddenly insulated and kept so. Find the temperature distribution.

97) A homogeneous rod of conducting material of length a has its ends kept at zero temperature. The temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature function $u(x, t)$.

98) The ends A and B of a rod l cm long have their temperatures kept at $30°\mathrm{C}$ and $80°\mathrm{C}$, until steady state conditions prevail. The temperature of the end B is suddenly reduced to $60°\mathrm{C}$ and that of A increased to $40°\mathrm{C}$. Find the temperature distribution of the rod after time t.

99) Obtain temperature distribution $y(x, t)$ in a uniform bar of unit length whose one end is kept at $10°\mathrm{C}$ and the other end is insulated. Further it is given that $y(x, 0) = 1 - x, 0 < x < 1$.

100) An insulated rod of length l has its ends A and B kept at $a°$ calsius and $b°$ celsius respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to zero degree celsius and kept so. Find the resulting temperature at any point of the rod taking the end A as origin.

101) If both the ends of a bar of length a are insulated and the initial temperature $f(x)$ is prescribed, then to find the temperature at a subsequent time t.

102) Solve $k(\partial^2 u/\partial x^2) = \partial u/\partial t$ for $0 < x < \pi, t > a$, if $u_x(0, t) = u_x(\pi, t) = 0$ and $u(x, 0) = \sin\, x$. [I.A.S. 2002]

Laplace

103) The Dirichlet problem in a rectangle is defined as follows:

Laplace's equation : $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, 0 \leq x \leq a,\ 0 \leq y \leq b$ Boundary conditions :

$$\begin{aligned} u(0, y)=0, \quad & u(a, y)=0, \quad 0 \leq y \leq b \\ u(x, b)=0, \quad & 0 \leq x \leq a \\ u(x, 0)=f(x), \quad & 0 \leq x \leq a \end{aligned}$$

104) Find the steady temperature distribution in a thin plate bounded by the lines $x=0, x=a, y=0$ and $y=\infty$ assuming that heat cannot escape from either surface; the sides $x=0, x=a$ being kept at temperature zero. The lower edge $y=0$ is kept at $f(x)$ and the edge $y=\infty$ at temperature zero

105) Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions. $u(0, y)=u(a, y)=0$ for $0 \leq y \leq b$ and $u(x, 0)=0$ and $u(x, b)=f(x)$ for $0 \leq y \leq a$. $u_y(x, b)=f(x),\ 0 \leq x \leq a$

106) The Neumann problem in a rectangle is defined as follows:

Laplace's equation:$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ 0 \leq x \leq a,\ 0 \leq y \leq b$ Boundary conditions:

$$\begin{aligned} & u_x(0, y)=u_x(a, y)=0,\ 0 \leq y \leq b \\ & u_y(x, 0)=0,\ 0 \leq x \leq a \quad u_y(x, b)=f(x), 0 \leq x \leq a \end{aligned}$$

107) Evaluate the steady temperature in a rectangular plate of length a and width b, the sides of which are kept at temperature zero, the lower end is kept at temperature $f(x)$ and the upper edge is kept insulated.

108) Find the steady state temperature distribution in a rectangular plate bounded by the lines $x=0, x=a, y=0$ and $y=b$ if the edge $y=0$ is insulated, the edges $x=0$ and $x=a$ are kept at $0^{\circ} C$ and edge $y=b$ is kept at temperature $f(x)$.

109) A rectangular metal plate is bounded by the lines $x=0, x=a, y=0$ and $y=b$. The three sides $x=0, x=a$ and $y=b$ are insulated and the side $y=0$ is kept at temperature $u_0 \cos(\pi x / a)$. Find the steady state temperature at any point of the plate.

Polar Equations

110) Solve the differential equation $(\partial^2 u / \partial r^2)+(1 / r) \times(\partial u / \partial r)+(1 / r^2) \times(\partial^2 u / \partial \theta)=0$ subject to the boundary conditions: (i) u is finite when $r \rightarrow 0$ (ii) $u=\Sigma C_n \cos\ n \theta$ when $r=a$.

111) A thin semi-circular plate of radius a has its boundary diameter kept at $0^{\circ}\mathrm{C}$ and its circumferene at $f(\theta)$. Find the temperature distribution in the steady state. A thin semi-circular plate of radius a has its boundary diameter kept at $0^{\circ}\mathrm{C}$ and its circumference at $100^{\circ}\mathrm{C}$. If $u(r,\theta)$ is the steady state temperature, find $u(a/4,\pi/2)$.

112) A semi-circular plate of radius a is kept at temperature u_0 along the bounding diameter and u_1 along the circumference. Find the steady state temperature at any point of the plate.

113) A circular sector is determined by $0 \leq r \leq a, 0 \leq \theta \leq \alpha$. The temperature is kept $0^{\circ}\mathrm{C}$ along the straight edges and at $f(\theta)$ along the curved edge. Find the steady state temperature at any point of the sector with its surface insulated.

114) Obtain steady temperature distribution in a semi-circular plate of radius a, insulated on both faces, with its curved boundary kept at a constant temperature u_0 and its boundary diameter kept at zero temperature.

115) u is a function of r and θ satisfying the equation. $\frac{\partial^2 u}{\partial r^2}+\left(\frac{1}{r}\right)\times\left(\frac{\partial u}{\partial r}\right)+\left(\frac{1}{r^2}\right)\times\left(\frac{\partial^2 u}{\partial \theta^2}\right)=0$ within the region of the plane bounded by $r=a, r=b, \theta=0, \theta=\pi/2$. Its value along the boundary $r=a$ is $\theta(\pi/2-\theta)$, and it value along the other boundary is zero. Prove that

$$u=\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{(r/b)^{4m-2}-(b/r)^{4m-2}}{(a/b)^{4m-2}-(b/a)^{4m-2}}\frac{\sin(4m-2)\theta}{(2m-1)^3}$$

116) Find solution of two-dimensional Laplace's equation $r^2(\partial^2 u/\partial r^2)+r(\partial u/\partial r)+(\partial^2 u/\partial \theta^2)=0$ in polar co-ordinates. or Solve heat equation in steady state in two-dimensional polar co-ordinates.

117) Consider a circular annulus of inner radius r_1 and outer radius r_2. Let the surface of the annulus be insulated. Find the steady state temperature at any point (r,θ) in the annulus, given that the temperature distribution along the inner circle $r=r_1$ and the outer circle $r=r_2$ are maintained as $u(r_1,\theta)=f_1(\theta)$ and $u(r_2,\theta)=f_2(\theta)$. A plate in the form of a ring is bounded by the circles $r=2$ and $r=4$. Its surfaces are insulated and the temperature $u(r,\theta)$ along the boundary are $u(2,\theta)=6\cos\theta+10\sin\theta$ and $u(4,\theta)=15\cos\theta+17\sin\theta$. Find the steady state temperature $u(r,\theta)$ in the ring.

118) Interior Dirichlet problem for a circle : Find the steady state temperature in a circular plate of radius a whose circumference is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface.

(a) Find the steady state temperature in a circular plate of radius a which has one half of its circumference at 0°C and the other half at constant temperature u_0C.
(b) A circular plate of radius a has half of its boundary kept at constant temperature u_1 and the other half at constant temperature u_2, where $u_1 \neq u_2$. Find the steady state temperature of the plate.

Cauchy Strips

119) Prove that for the equation $z + px + qy - 1 - pqx^2y^2 = 0$ the characteristic strips are given by $x(t) = \frac{1}{B+Ce^{-t}}, y(t) = \frac{1}{A+De^{-t}}, z(t) = E - (AC + BD)e^{-t}$ $p(t) = A(B + Ce^{-t})^2, q(t) = B(A + De^{-t})^2$ where A, B, C, D and E are arbitrary constants. Hence find values of these arbitrary constants if the integral surface passes through the line $z = 0, x = y$

120) Solve the first order quasi linear PDE by the method of characteristics: $x\frac{\partial u}{\partial x} + (u - x - y)\frac{\partial u}{\partial y} = x + 2y$ in $x \geq 0, -\infty < y < \infty$ with $u = 1 + y$ on $x = 1$

121) Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes though the parabola $4z + x^2 = 0$

122) Solve the following partial differential equation

$$zp + yq = x$$
$$x_0(s) = s, y_0(s) = 1, z_0(s) = 2s$$

by the method of characteristics.

123) Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \frac{x}{2}, y = 0$

124) Find characteristic of pq = z and surface passing through x = 1, z = y.

125) Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the parabola $x = 0, y^2 = z$.

126) Find the solution of $z = (p^2 + q^2)/2 + (p - x)(q - y)$ which passes through the x-axis.

127) Find the characteristics of the partial differential equation $p^2 + q^2 = 2; p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$ and determine the integral surface which passes through $x = 0, z = y$.

Extra Questions

128) Reduce the following partial differential equation to a canonical form and hence solve it: $yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0$

129) A thin annulus occupies the region. $0 \leq a \leq r \leq b, 0 \leq \theta \leq 2\pi$ The faces are insulated, Along the inner edge the temperature is maintained $0°$, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$ where K is a constant. Determine the temperature distribution in the annulus.

130) Find the partial differential equation of the family of all tangent planes to the ellipsoid $x^2 + 4y^2 + 4z^2 = 4$ which are not perpendicular to the xy plane.

131) Find the general solution of the partial differential equation:
$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1$.