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UPSC CSE Mathematics: Previous Year Questions: Statics

2025

1) Given that A and B are two points in the same horizontal line distant 2a apart. AO and BO are two equal heavy strings tied together at O and carrying their weight at O . If l is length of each string and d is depth of O below AB, then show that the parameter c of this catenary, in which the strings hang, is given by $l^2 - \mathrm{d}^2 = 2\mathrm{c}^2\left[\cosh\left(\frac{\mathrm{a}}{\mathrm{c}}\right) - 1\right]$

2) A solid sphere rests inside a fixed rough and hemispherical bowl of twice its radius. If a large amount of weight, whatsoever, is attached to the highest point of the sphere, then show that the equilibrium is stable.

2024

1) A solid hemisphere rests in equilibrium on a solid sphere of equal radius. Determine the stability of the equilibrium in the two situations (i) when the curved surface and (ii) when the flat surface of the hemisphere rests on the sphere.

2) A regular tetrahedron, formed of six light rods, each of length l, rests on a smooth horizontal plane. A ring of weight W and radius r is supported by the slant sides. Using the principle of virtual work, find the stress in any of the horizontal sides.

3) A heavy particle hanging vertically from a fixed point by a light inextensible string of length l starts to move with initial velocity u in a circle so as to make a complete revolution in a vertical plane. Show that the sum of tensions at the lends of any diameter is constant.

2023

1) A cylinder of radius ' a ' touches a vertical wall along a generating line. Axis of the cylinder is fixed horizontally. A uniform flat beam of length ' l ' and weight ' W ' rests with its extremities in contact with the wall and the cylinder, making an angle of $45°$ with the vertical. If frictional forces are neglected, then show that $\frac{a}{l} = \frac{\sqrt{5}+5}{4\sqrt{2}}$ Also, find the reactions of the cylinder and wall.

2) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface is in contact. If θ is the angle of inclination of the string with vertical and ϕ is the angle of inclination of the plane base of the hemisphere to the vertical, then find the value of $(\tan\ \phi - \tan\ \theta)$.

2022

1) A cable of weight w per unit length and length $2l$ hangs from two points P and Q in the same horizontal line. Show that the span of the cable is $2l\left(1-\frac{2\,\mathrm{h}^2}{3l^2}\right)$, where h is the sag in the middle of the tightly stretched position.

2) Suppose a cylinder of any cross-section is balanced on another fixed cylinder, the contact of curved surfaces being rough and the common tangent line horizontal. Let ρ and ρ' be the radii of curvature of the two cylinders at the point of contact and h be the height of centre of gravity of the upper cylinder above the point of contact. Show that the upper cylinder is balanced in stable equilibrium if $\mathrm{h} < \frac{\rho\rho'}{\rho+\rho'}$.

3) A chain of n equal uniform rods is smoothly jointed together and suspended from its one end A_1. A horizontal force $\vec{\mathrm{P}}$ is applied to the other end A_{n+1} of the chain. Find the inclinations of the rods to the downward vertical line in the equilibrium configuration.

2021

1) Two rods LM and MN are joined rigidly at the point M such that $(LM)^2 + (MN)^2 = (LN)^2$ and they are hanged freely in equilibrium from a fixed point L. Let ω be the weight per unit length of both the rods which are uniform. Determine the angle, which the $\mathrm{rod}\ LM$ makes with the vertical direction, in terms of lengths of the rods.

2) A heavy string, which is not of uniform density, is hung up from two points. Let T_1, T_2, T_3 be the tensions at the intermediate points A, B, C of the catenary respectively where its inclinations to the horizontal are in arithmetic progression with common difference β. Let ω_1 and ω_2 be the weights of the parts AB and BC of the string respectively. Prove that

   (i) Harmonic mean of T_1, T_2 and $T_3 = \frac{3T_2}{1+2\cos\,\beta}$

   (ii) $\frac{T_1}{T_3} = \frac{\omega_1}{\omega_2}$

2020

1) A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest?

2) A beam AD rests on two supports B and C, where $AB = BC = CD$. It is found that the beam will tilt when a weight of is hung from or when a weight of p kg is hung from A or when a weight of q kg is hung from D. Find the weight of the beam.

3) A square framework formed of uniform heavy rods of equal weight W jointed together, is hung up by one corner. A weight W is suspended from each of the three lower corners, and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod.

2019

1) One end of a heavy uniform rod AB can slide along a rough horizontal rod AC to which it is attached by a ring. B and C are joined by a string. When the rod is on the point of sliding then $AC^2 - AB^2 = BC^2$. If θ is the angle between AB and the horizontal line, then prove that the coefficient of friction is $\frac{\cot\theta}{2+\cot^2\theta}$

2) A body consists of cone and underlying hemisphere. The base of the cone and the top of the hemisphere have same radius a. The whole-body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}a$

2017

1) A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle ϕ with its curve surface touching the plane. Find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable?

2016

1) A uniform rod AB of length $2a$ movable about a hinge at A rests with other end against a smooth vertical wall. If α is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is $\frac{1}{2}W\sqrt{4+\tan^2\alpha}$ where W is the weight of the rod.

2) Two weights P and Q are suspended from a fixed point O by strings OA, OB and are kept apart by a light rod AB if the strings OA and OB make angle α and β with the rod AB show that the angle θ which the rod makes with the vertical is given by $\tan\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}$

3) A square $ABCD$ the length of whose side is a is fixed in a vertical plane with two of its sides horizontal An endless string of length $l(>4a)$ passes over four pegs at the angle of the board and through a ring of weight W which is hanging vertically show that the tension of the string is $\frac{W(l-3a)}{(\sqrt{l^2-6la+8a^2})}$.

2015

1) A rod of 8 kg is movable in a vertical plane about a hinge at one end another end is fastened a weight equal to half of the rod, this is fastened by a string of length l to a point at a height to above the hinge vertically. Obtain the tension in the sting.

2) Two equal ladders of weight 4 kg each are placed so as to lean at A against each other with their end resting on a rough floor, given the coefficient of friction is μ. The ladders at A make an angle $60°$ with each other. Find what weight on the top would cause them to slip.

3) Find the length of an endless chain which will hang over circular pulley of radius ' a ' so as to be in contact with the two thirds of the circumference of the pulley.

2014

1) Two equal uniform rods AB and AC, each of length l are freely jointed at A and rest on a smooth fixed vertical circle of radius r. If 2θ is the angle between the roads, then find the relation between l, r and θ by using the principal of virtual work.

2) A regular pentagon $ABCDE$, formed of equal heavy uniform bars jointed together is suspended from the joint A, and is maintained in from by a light road joining the middle points of BC and DE. Find the stress in this rod.

2013

1) The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight.

2) A uniform ladder rests at an angle of $45°$ with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If μ and μ ' are the coefficients of limiting friction between the ladder and the ground and wall respectively, the find the minimum horizontal force required to move the lower end of the ladder towards the wall.

3) Six equal rods AB, BC, CD, DE, EF and FA are each of weight w and are freely jointed at their extremities so as to from a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by string. Find the tension in the string.

2012

1) A heavy hemispherical shell of radius a has a particle attached a point on the rim, and rests with the curved surface in contact with a rough sphere of radius b at the highest point. Prove that it $\frac{b}{a} > \sqrt{5} - 1$ the equilibrium is stable, whatever be the weight of the particle.

2) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log \left[\frac{1+\sqrt{1+\mu^2}}{\mu}\right]$, where μ the coefficient of friction is.

2011

1) A ladder of weight W rests with one end against a smooth vertical wall and the other end rest on a smooth floor. If the inclination of the ladder to the horizon is $60°$, find the horizontal force that they must be a applied to the lower end to prevent the ladder from slipping down.

2010

1) Solid hemisphere is supported by a string fixed to point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclination of the string and the plane base of the hemisphere to the vertical, prove by using the principal of virtual work that $\tan\ \phi = \frac{3}{8} + \tan\ \theta$

2009

1) A uniform rod AB is movable about a hinge at A and rests with one end in contact with a smooth vertical wall. If the rod is inclined at an angle of $30°$ with the horizontal, find the reaction at the hinge in magnitude and direction.
2) Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with three-fourth of the circumference of the pulley.

2008

1) A straight uniform beam of length ' $2h$ ' rests in limiting equilibrium in contact with a rough vertical wall of height ' h ' with one end on a rough horizontal plane and with the other end projecting beyond the wall If both the wall and the plane be equally rough, prove that ' λ ' the angle of friction, is given by $\sin\ 2\lambda = \sin\ \alpha \sin\ 2\alpha$,' α' being the inclination of the beam to the horizon.
2) A ladder of weight 10 kg. rests on a smooth horizontal ground leaning against a smooth vertical wall at an inclination $\tan^{-1}\ 2$ with the horizontal and is prevented from slipping by a string attached at it lower end and to the junction of the floor and the wall A body of weight 30 kg begins to ascend the ladder. If the string can bear a tension of 10 kg. wt., how far along the ladder cans the boy rise with safety?
3) A solid right circle cone whose height is h and radius of whose base is r, is placed on an inclined plane and it is prevented from sliding. If the inclination θ of the plane (to the horizontal) be gradually decreased, find when the cone will topple over. For a cone whose semi- vertical angle is $30°$ determine the circular value of θ which when exceeded, the cone will topple over.

2007

1) A uniform string of length one-meter hangs over two smooth pegs P and Q at different heights. The parts which hand vertical are of length 34 cm and 26 cm. find the ratio in which the vertex of the Catenary divides the whole string.

2) A uniform beam of length l rests with its ends on two smooth planes which intersect in a horizontal line if the inclination of the planes to the horizontal are α and $(\beta > \alpha)$, show that the inclination θ of the beam to the horizontal, in one of the equilibrium positions, is given by $\tan\ \theta = \frac{1}{2}(\cot\ \alpha - \cot\ \beta)$, and show that the beam is unstable in this position.

2006

1) The middle point of opposite sides of a jointed quadrilateral are connected by light rods of length l, l'. if T, T' be the tension in these rods, prove $\frac{T}{l} + \frac{T'}{l'} = 0$.

2) Show that the length of an endless chain which will hang over a circular pulley of radius C so as to be in contact with two third of the circumference of the pulley is $c\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$.

2005

1) If a number of concurrent forces be represented in magnitude and direction by the sides of a closed polygon, taken in other then show that these forces are in equilibrium.

2) Two equal uniform rods AB and , of length a each , are freely joined at A, and are placed symmetrically over two smooth pegs on the same horizontal level at a distance c apart $(3c < 2a)$. A weight equal to the of a road, is suspended from the joint A .In the position of equilibrium, find the inclination of the either rod with the horizontal by the principal of virtual work.

2004

1) A non-uniform string hangs under gravity. Its cross-section at any point is inversely proportion to the tension at that point. Prove that the curve in which the string hangs is an arc of a parabola with its axis vertical.

2) A uniform bar AB weights $12N$ and rests with one part AC of length 8 m, on a horizontal table and the remaining part CB projecting over the edge of the table if the bar is on the point of overbalancing when a weight of 5 N is placed on it at point 2 m from A and a weight of 7 N is hung from B find the length of AB.

2003

1) A sphere of weight W and radius a lies within a fixed spherical shell of radius b. A particle of weight w is fixed to the upper end of the vertical diameter. Prove that equilibrium is stable if $\frac{W}{w} > \frac{b-2a}{a}$.

2) A ladder on a horizontal floor lean against a vertical wall the coefficients of friction of the floor and the wall with the ladder are μ and μ ' respectively If a man, whose weight is n times that of the ladder, wants to climb up the ladder, find the minimum safe angle of the ladder with the floor.

2002

1) Obtain the equation of the curve in which a string hangs under gravity from two fixed points (not lying in a vertical line) when line mass density at each of its points varies as the radius of curvature of the curve.

2) Five weightless rods of equal length are jointed together so as to from a rhombus $ABCD$ with a diagonal BD. If a weight W be attached to C and the system be suspended from a point A, show that the thrust in BD is equal to $\frac{W}{\sqrt{3}}$.

2001

1) The middle points of opposite sides of a jointed quadrilateral are connected by light rods of length l, l'. If T, T' be the tension in these rods, prove $\frac{T}{l} + \frac{T'}{l'} = 0$.

2) OA, OB and OC are edges of side a, and OO', AA', BB' and CC' are its diagonals. Along $OB', O'A, BC$ and $C'A'$ act force equal to $P, 2P, 3P$ and $4P$ respectively. Reduce the system to a force at O together with a couple.