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UPSC CSE Mathematics: Previous Year Questions: Dynamics

2025

1) Prove that the time taken by the Earth to travel over half of its orbit, which is separated by the minor axis and is remote from the Sun, when the Sun is at the focus of the elliptic orbit, is two days more than half of the year. The eccentricity of the orbit is taken as $\frac{1}{60}$.

2) One end of an elastic string, having natural length a, is fixed at some point O and a heavy particle is attached to the other end of the string. The string is drawn vertically downward till it is four times its natural length at the point C and then released. If the modulus of elasticity of the string is equal to the weight of the particle, then show that the particle will return to the same point C in the time $\sqrt{\frac{a}{g}}\left(2\sqrt{3}+\frac{4\pi}{3}\right)$.

3) A particle is projected inside a fixed smooth cylinder with circular cross-section in a vertical plane from the lowest point with initial horizontal velocity u. Show that for
(i) $(u^2 \leq 2ag)$; the particle oscillates about the mean position in the lower half,
(ii) $(u^2 \geq 5ag)$; the particle executes complete circular motion, and
(iii) $(2ag < u^2 < 5ag)$; the particle will leave the curve in a tangential direction, making an angle α with the horizontal such that $\cos\ \alpha = \frac{u^2-2ag}{3ag}$.

2024

1) A particle moves with a central acceleration $\mu\left(\frac{3}{r^3}+\frac{d^2}{r^5}\right)$ being projected from a distance d at an angle $45°$ with a velocity equal to that in a circle at the same distance. Prove that the time it takes to reach the centre of force is $\frac{d^2}{\sqrt{2\mu}}\left(2-\frac{\pi}{2}\right)$

2) A particle executes simple harmonic motion such that in two of its positions, velocities are u and v, and the two corresponding accelerations are f_1 and f_2. For what value(s) of k, the distance between the two positions is $k(v^2-u^2)$? Show also that the amplitude of the motion is $\frac{1}{f_2^2-f_1^2}[(u^2-v^2)(u^2f_2^2-v^2f_1^2)]^{1/2}$

2023

1) A particle is projected from an apse at a distance $\sqrt{c}$ from the centre of force with a velocity $\sqrt{\frac{2\lambda}{3}}c^3$ and is moving with central acceleration $\lambda(r^5 - c^2 r)$. Find the path of motion of this particle. Will that be the curve $x^4 + y^4 = c^2$?

2) A particle is moving under Simple Harmonic Motion of period T about a centre O. It passes through the point P with velocity v along the direction OP and $OP = p$. Find the time that elapses before the particle returns to the point P. What will be the value of p when the elapsed time is $\frac{T}{2}$?

3) When a particle is projected from a point O_1 on the sea level with a velocity v and angle of projection θ with the horizon in a vertical plane, its horizontal range is R_1. If it is further projected from a point O_2, which is vertically above O_1 at a height h in the same vertical plane, with the same velocity v and same angle θ with the horizon, its horizontal range is R_2. Prove that $R_2 > R_1$ and $(R_2 - R_1): R_1$ is equal to $\frac{1}{2}\left\{\sqrt{\left(1 + \frac{2gh}{v^2 \sin^2 \theta}\right)} - 1\right\}: 1$

2022

1) A body of weight w rests on a rough inclined plane of inclination θ, the coefficient of friction, μ, being greater than $\tan \theta$. Find the work done in slowly dragging the body a distance 'b' up the plane and then dragging it back to the starting point, the applied force being in each case parallel to the plane.

2) A projectile is fired from a point O with velocity $\sqrt{2\mathrm{gh}}$ and hits a tangent at the point $\mathrm{P}(\mathrm{x}, \mathrm{y})$ in the plane, the axes OX and OY being horizontal and vertically downward lines through the point O, respectively. Show that if the two possible directions of projection be at right angles, then $\mathrm{x}^2 = 2\mathrm{hy}$ and then one of the possible directions of projection bisects the angle POX.

2021

1) If a planet, which revolves around the Sun in a circular orbit, is suddenly stopped in its orbit, then find the time in which it would fall into the Sun. Also, find the ratio of its falling time to the period of revolution of the planet.

2) A heavy particle hangs by an inextensible string of length a from a fixed point and is then projected horizontally with a velocity $\sqrt{2gh}$. If $\frac{5a}{2} > h > a$, then prove that the circular motion ceases when the particle has reached the height $\frac{1}{3}(a + 2h)$ from the point of projection. Also, prove that the greatest height ever reached by the particle above the point of projection is $\frac{(4a-h)(a+2h)^2}{27a^2}$.

3) Describe the motion and path of a particle of mass m which is projected in a vertical plane through a point of projection with velocity u in a direction making an angle θ with the horizontal direction. Further, if particles are projected from that point in the same vertical plane with velocity $4\sqrt{g}$, then determine the locus of vertices of their paths.

2020

1) A light rigid rod ABC has three particles each of mass m attached to it at A, B and C the rod is struck by a blow P at right angles to it at a point distance from A equal to BC. Prove that the kinetic energy set up is $\frac{1}{2}\frac{P^2}{m}\frac{a^2-ab+b^2}{a^2+ab+b^2}$ where $AB = a$ and $BC = b$

2) A particle starts at a great distance with velocity V. let p be the length of the perpendicular from the centre of a star on the tangent to the initial path of the particle. Show that the least distance of the particle from the centre of the star is λ, where $V^2\lambda = \sqrt{\mu^2 + p^2V^4} - \mu$. Here μ is a constant.

3) A four-wheeled railway truck has a total mass M, the mass and radius of gyration of each pair of wheels and axel m are k respectively, and the radius of each wheel is r. Prove that if the truck is propelled along a level track by a force P, the acceleration is $\frac{P}{M+\frac{2mk^2}{r^2}}$, and find the horizontal force exerted on each axel by the truck. The axle friction and wind resistance are to be neglected.

2019

1) The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's centre. A Particle, whose weight on the surface of the earth is W, falls to the surface of the earth from a height $3h$ above it. Show that the magnitude of work done by the earth s attraction force is $\frac{3}{4}hW$, where h is the radius of the earth.

2) A particle moving along the y-axis has an acceleration Fy towards the origin, where F is a positive and even function of y. The periodic time, when the particle vibrated between $y = -a$ and $y = a$ is T. Show that $\frac{2\pi}{\sqrt{F_1}} \leq T \leq \frac{2\pi}{\sqrt{F_2}}$ where F_1 and F_2 are the greatest and the least values of F within the range $[-a, a]$. Further show that when a simple pendulum of length l oscillates through $30°$ on either side of the vertical line, T lies between $2\pi\sqrt{l/g}$ and $2\pi\sqrt{l/g}, \sqrt{\pi/3}$

3) Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to $\frac{\mu}{(\text{distance})^2}$, is a conic section. Find the conditions under which the path becomes (i) ellipse (ii) parabola and (iii) hyperbola.

2018

1) A particle projected from a given point on the ground just clears a wall of height h at a distance d from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R find elevation of the projection.

2) A particle moving with simple harmonic motion in a straight line has velocities v_1 and v_2 at distances x_1 and x_2 respectively from the centre of its path. Find the period of its motion.

2017

1) A fixed wire is in the shape of the Cardioid $r = a(1 + \cos\ \theta)$, the initial line being the downward vertical. A small ring of mass m can slide on the wire and is attached to the point $r = 0$ of the Cardioid by an elastic string of natural length a and modulus of elasticity 4mg. The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that $a\theta^2(1 + \cos\ \theta) - g\cos\ \theta(1 - \cos\ \theta) = 0, g$ being the acceleration due to gravity.

2) A particle is free to move on a smooth vertical circular wire of radius a. At time $t = 0$ is projected along the circle from its lowest point A with velocity just sufficient to carry it to the highest point B. Find the time T at which the reaction between the particle and the wire is zero.

3) A spherical shot of W gm weight and radius r cm, lies at the bottom of cylindrical bucket of radius R cm. The bucket is filled with water up to a depth of $h\,\text{cm}(h > 2r)$. Show that the minimum amount of work done in lifting the shot just clear of the water must be $\left[W\left(h - \frac{4r^3}{3R^2}\right) + W'\left(r - h + \frac{4r^3}{3R^2}\right)\right]$ cm gm. W' gm is the weight of water displaced by the shot.

2016

1) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is Projected from an apse at a distance a from the origin with a velocity which $\sqrt{2}$ is times the velocity for a circle or radius a then find the equation to the path.

2) A particle moves in a straight line. Its acceleration is directed towards a fixed point O in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{1/3}$ when it is at a distance x from O. If it starts rest at a distance a form O, then find the time the particle will arrive at O.

2015

1) A body moving under SHM has an amplitude a and time period T, If velocity is trebled, when the distance from mean position is $\frac{2}{3}a$, the period being unaltered find the new amplitude.

2) A mass m starts from rest at a distance ' a ' from the center of force which attract inversely as the distance find the time of arriving at the center.

3) A particle is projected from the base of a bill whose slope is that of a right circular cone whose axis is vertical. The projectile grazes the vertex and strikes the hill again at point on the base. if the semi vertical angle of the cone is $30°\text{h}$ is height determine the initial velocity u of the projection and its angle of projection.

4) A particle moves in a plane a force towards a fixed centre proportional to the distance. If the path of the particle has apsidal distance $a, b(a > b)$, then find the equation of the path.

2014

1) A particle is performing a simple harmonic motion (S.H.M) of period T about a centre O with amplitude a and it passes through a points P where $OP = b$ in the direction OP. Prove that the time which elapse before it returns to P is $\frac{T}{\pi}\cos^{-1}\left(\frac{b}{a}\right)$.

2) A particle of mass m, hanging vertically from a fixed point by a light inextensible cord of length l is struck by a horizontal blow which imparts of it a velocity $2\sqrt{\text{gl}}$. Find the velocity and height of the particle from of its initial position when the cord becomes slack.

3) A particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μy^{-2} and when $y = a$ it is projected parallel to the axis of x with velocity $\sqrt{\frac{2\mu}{a}}$. Find the parametric equation on the path of the particle. Here μ is constant.

2013

1) A body is performing S.H.M in a straight line OPQ. Its velocity is zero at points P and Q whose distances from O are x and y respectively and its velocity is v at the mid-point between P and Q. Find the time of one complete oscillation.

2) A particle of mass 2.5 kg hangs at the end of a string 0.9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward.

2012

1) A particle moves with an acceleration $\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a from the origin, find its velocity when its distance from the origin is $\frac{a}{2}$.

2) A heavy ring of mass m, slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distance a from the rod and has a mass $M(> m)$ fastened to its order end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley it will descend a distance $\frac{2Mma}{M^2-m^2}$ before coming to rest.

2011

1) The velocity of a train increases from 0 to v at constant acceleration f_1 then remains constant for an interval and again decreases to 0 at a constant retardation f_2' If the total distance described is x find the total time taken.

2) A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a meter short of it when the angle of projection is α and goes y meter beyond w then the angle of projection is β. If the velocity of projection is assumed same in all cases, find the correct angle of projection.

3) A mass of 560 kg. moving with a velocity of 240 m/sec strikes a fixed target and is brought to rest in $\frac{1}{100}$ sec. Find the impulse of the blow on the target and assuming the resistance to be uniform throughout the time taken by the body in coming to rest, find the distance through which it penetrates.

4) After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity if it now reaches the ground in 1 second, find the height of glass above the ground.

5) A particle of mass m moves on straight line under an attractive force mn^2x towards a point O on the line, where x is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find $x(t)$ for any time $t > 0$

2010

1) If v_1, v_2, v_3 are the velocities at three points A, B, C of the path of a projectile, where the inclinations to the horizon are $\alpha, \alpha - \beta, \alpha - 2\beta$ and if t_1, t_2 are the times of describing the arcs AB, BC respectively. Prove that $v_3t_1 = v_1t_2$ and $\frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$

2) A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half.

3) A particle moves with a central acceleration $\mu(r^5 - 9r)$ being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{2\mu}$. Show that the path is the curve $x^4 + y^4 = 9$.

2009

1) A body is describing an ellipse of eccentricity e under the action of a central force directed towards a focus and when at the nearer apse, the center of force is transferred to the other focus. Find the eccentricity of the new orbit in terms of the eccentricity of the original orbit.

2) A shot fired with a velocity V at an elevation α strikes a point P in a horizontal plane through the point of projection. If the point P is receding from the gun with velocity v, show that the elevation must be changed to θ where $\sin 2\theta = \sin 2\alpha + \frac{2v}{V}\sin\theta$

3) One end of light elastic of natural length l and modulus of elasticity 2mg is attached to fixed point O and the other end to a particle of mass m, the particle initially held at rest at O is let fall. Find the greatest extension of the string during the motion and show that the particle will reach 0 again after a time. $(\pi + 2 - \tan^{-1}\ 2)\sqrt{\frac{21}{g}}$

4) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down are arc. Show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\cot^{-1}\left(\frac{V}{2\sqrt{ag}}\right)$

2008

1) A smooth parabolic tube is placed with vertex downwards in vertical plane. A particle slides down the tube from rest under the influence of gravity. prove that in any position, the reaction of the tube is equal to $2w\left(\frac{h+a}{\rho}\right)$, where ' w ' is the weight of the particle , ' p ' the radius of curvature of the tube , ' $4a'$ 'its latus rectum and ' h ' the initial vertical height of the particle above the vertex of the tube.

2) A particle P moves in a plane such that it is acted on by two constant velocities u and v respectively along the direction OX and along the direction perpendicular to OP where O is same fixed point, that is the origin. Show that the path traversed by P is a conic section with focus at O and eccentricity $\frac{u}{v}$.

3) A particle of mass m moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\},\ u = \frac{1}{r}, a > b, a, b$ and $\mu(> 0)$ being given constants. It is projected from an apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is given by the equation $r = a + b\cos\ \theta$, where (r, θ) are the plane polar coordinates of a point.

4) A shell lying in a strength smooth horizontal tube suddenly breaks into two portions of masses m_1 and m_2 if s be the distance between the two masses inside the tube after time t, show that the work done by the explosion can be written as equal to $\frac{m_1 m_2}{m_1+m_2}\frac{s^2}{t^2}$.

2007

1) A particle falls from rest under gravity in a medium whose resistance varies as the velocity of the particle. Find the distance fallen by the particle and its velocity at time t.

2) A particle is performing simple harmonic motion of period T about a centre O. It passes through a point $P(OP = p)$ with velocity v in the direction OP. Show that the time which elapses before it returns to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi p}\right)$.

3) A particle attached to a fixed peg O by a string of length l , is lifted up with the string horizontal and then let go. Prove that when the string makes an angle θ with the horizontal the resultant acceleration is $g\sqrt{(1+3\sin^2\ \theta)}$

2006

1) A particle is free to move on a smooth vertical circular wire of radius a. it is projected horizontal from the lowest point with velocity $2\sqrt{ga}$. Show that the reaction between the particle and the wire is zero after is time $\sqrt{\frac{a}{g}}\log\left(\sqrt{5}+\sqrt{6}\right)$

2) A particle, whose mass in m is acted upon by a force $m\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at origin in time $\frac{\pi}{4}$.

3) If u and v are the velocity of projection and the terminal velocity respectively of a particle rising vertically against a resistance varying as the square of the velocity, prove that the time taken by the particle to reach the highest point is $\frac{v}{g}\tan^{-1}\left(\frac{u}{v}\right)$.

2005

1) A body of mass (m_1+m_2) moving in a straight line is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E. If after the explosion, the two parts move in the same line as before, find their relative velocity.

2) A particle is projected along the inner side of a smooth vertical of radius a so that its velocity particle at the lowest point is u. Show that if $2ag<u^2<5ag$,the particle will leave the circle before arriving at the highest point and will describe a parabola whose latus rectum is $\frac{2\left(u^2-2ga\right)^3}{27g^3a^2}$.

3) Two particles connected by a fine string are constrained to move in a fine cycloid tube in a verticals plane. The axis of the cycloid is vertical with vertex upwards. Prove that tension in the sting is constant throughout the motion.

2004

1) A point moving with uniform acceleration describes distances s_1 and s_2 metres in successive intervals of time t_2 and t_2 seconds. Express the acceleration in terms of s_1, s_2, t_1 and t_2.

2) Prove that the velocity required to project a particle from a height h to fall at a horizontal distance a from a point of projection, is at least equal to $\sqrt{g\left[\sqrt{a^2+h^2}-h\right]}$.

3) A car of mass $750\ \text{kg}$ is running up a hill of 1 in 30 at a steady speed of $36\ \text{km/hr}$; the friction is equal to the weight of $40\ \text{kg}$. Find the work done in 1 second.

4) A uniform bar AB weights $12N$ and rests with one part AC of length 8 m, on a horizontal table and the remaining part CB projecting over the edge of the table. If the bar is on the point of overbalancing when a weight of 5 N is placed on it at point 2 m from A and a weight of 7 N is hung from , find the length of AB.

2003

1) A particle describes the curve $r = a(1 + \cosh\ \theta)/(\cosh\ \theta - 2)$ under a force F to the pole. Show that the law of force is $F \propto \frac{1}{r^4}$.

2) An elastic string of natural length $a + b$, where $a > b$ and modulus of elasticity λ has particle of mass m attached to it at a distance a from one end which is fixed to a point A of a smooth horizontal plane. The other end of the string is fixed to a point B so that the string is just unstretched. If the particle be held at B and then released find the periodic time and the distance in which the particle will oscillate to and fro.

3) If a particle slides down a smooth cycloid, starting from a point whose actual distance from the vertex is b, prove that its speed at any time t is $\frac{2\times b}{T}\sin\left(\frac{2xt}{T}\right)$ where T is the time of complete oscillation of the particle.

2002

1) A particle, whose mass is m is acted upon by a force $m\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, from the origin. Show that the time taken by it to reach the origin is $\frac{\pi}{4}$.

2) A heavy particle of mass m slides on a smooth arc of a cycloid in a medium whose resistance is $\frac{mv^2}{2c}$, v being the velocity of the particle and c being the distance of the starting point from the vertex. If the axis is vertical and vertex upwards, find the velocity of the particle at the cusp.

3) A particle describes a curve with constant velocity and its angular velocity about a given point O varies inversely as its distance from O. Show that the curve is an equiangular spiral.

2001

1) Find the law of force to the pole when the path of a particle is the cardioid $r = a(1 - \cos\ \theta)$ and prove that if F be the force at the apse and v the velocity there, then $3v^2 = 4aF$

2) A comet describing a parabola under inverse square law about the sun, when nearest to it suddenly breaks up, without gain or loss of kinetic energy, into two equal portions one of which describes a circle. Prove that that the other will describe a hyperbola of eccentricity 2.

3) A particle of mass M is at rest and begins to move under the action of a constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite

direction with velocity V which deposits matter on it at a constant rate ρ .Show that the mass of the particle will be m when it has travelled a distance $\frac{k}{\rho^2}\left[m - M\left\{1 + \log_e \frac{m}{M}\right\}\right]$ where $k = F - \rho V$.