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UPSC CSE Mathematics: Previous Year Questions: PDE

2025

1) Find the solution of the equation $(D^2 + DD' - 2D^2)z = y\sin\ x$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

2) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for a rectangular plate subject to the boundary conditions

$$u(0,y) = 0, u(a,y) = 0$$
$$u(x,0) = 0, u(x,b) = f(x)$$

3) Find the complete integral of $z(p^2 - q^2) = x - y; p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$.

4) Find the characteristics of the partial differential equation $p^2 + q^2 = 2; p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$ and determine the integral surface which passes through $x = 0, z = y$.

2024

1) Show that if f and g are arbitrary functions of their respective arguments, then $u = f(x - kt + i\alpha y) + g(x - kt - i\alpha y)$, is a solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}$, where $\alpha^2 = 1 - \frac{k^2}{C^2}$.

2) Show that the solution of the two-dimensional Laplace's equation

$$\frac{\partial^2 \phi(x,y)}{\partial x^2} + \frac{\partial^2 \phi(x,y)}{\partial y^2} = 0,\ x \in (-\infty, \infty), y \geq 0$$

subject to the boundary condition $\phi(x,0) = f(x), x \in (-\infty, \infty)$, along with $\phi(x,y) \to 0$ for $|x| \to \infty$ and $y \to \infty$ can be written in the form $\phi(x,y) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{f(\xi)d\xi}{y^2 + (x-\xi)^2}$

Out of Syllabus Question./ Fourier Transformation NOT in syllabus.

3) Find the integral surface of the following quasi-linear equation

$$(\mathbf{y} - \phi)\frac{\partial \phi}{\partial \mathbf{x}} + (\phi - x)\frac{\partial \phi}{\partial \mathbf{y}} = x - y,$$

which passes through the curve $\phi = 0, xy = 1$ and through the circle $x + y + \phi = 0, x^2 + y^2 + \phi^2 = a^2$. *Wrong/ Inconsistent Question : You need only one condition / Both invalid.*

4) Solve the partial differential equation $\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x} + \phi\right) + 2x^2 y\left(\frac{\partial \phi}{\partial x} + \phi\right) = 0$ by transforming it to the canonical form. *Highly Difficult to Solve by Canonical Method*

2023

1) By eliminating the arbitrary functions f and g from $z = f(x^2 - y) + g(x^2 + y)$, form partial differential equation.

2) Find the surface passing through the two lines $z = x = 0$ and $z - 1 = x - y = 0$. and satisfying the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x\, \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$.

3) Solve the partial differential equation $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < L, t > 0$ subject to the conditions

$$u(0, t) = 0,\ u(L, t) = 0,\ t > 0$$
$$u(x, 0) = x, \left(\frac{\partial u}{\partial t}\right)_{t=0} = 1,\ 0 < x < L$$

4) Reduce the PDE to canonical form $\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\left(1 + \frac{1}{x}\right) + \frac{z}{x} = 0$

2022

1) It is given that the equation of any cone with vertex at (a, b, c) is $f\left(\frac{x-a}{z-a}, \frac{y-b}{z-c}\right) = 0$. Find the differential equation of the cone.

2) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < l, t > 0$ subject to the conditions

$$u(0, t) = u(l, t) = 0$$
$$u(x, 0) = x(l - x), 0 \le x \le l$$

3) Find the general solution of $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$

4) Reduce the following partial differential equation to a canonical form and hence solve it: $yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0$

2021

1) Obtain the partial differential equation by eliminating arbitrary function f from the equation $f(x + y + z, x^2 + y^2 + z^2) = 0$.

2) Solve the wave equation $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < L,\ t > 0$ subject to the conditions

$$u(0, t) = 0, u(L, t) = 0$$
$$u(x, 0) = \frac{1}{4}x(L - x), \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$

3) Find the general solution of PDE $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

4) Find a complete integral of partial differential equation $p = (z + qy)^2$ by using Charpit's method.

2020

1) Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z = yf(x) + xg(y)$ and specify its nature (elliptic, hyperbolic or parabolic) in the region $x \geq 0, y \geq 0$

2) Solve the PDE: $(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{2x+y} + \sin(x - 2y)$;

3) Find the integral surface of the partial differential equation: $(x-y)y^2\frac{\partial z}{\partial x} + (y-x)x^2\frac{\partial z}{\partial y} = (x^2+y^2)z$ that contains the curve $xz = a^3, y = 0$ on it.

4) Find the solution of the partial differential equation: $z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ which passes through the x-axis.

5) One end of a tightly stretched flexible thin string of length l is fixed at the origin and the other at $x = l$. It is plucked at $x = \frac{l}{3}$ so that it assumes initially the shape of a triangle of height h in the $x - y$ plane. Find the displacement y at any distance x and at any time t after the string is released from rest. Take, $\frac{\text{horizontal tension}}{\text{mass per unit length}} = c^2$.

2019

1) Form a partial differential equation of the family of surface given by the following expression. $\psi(x^2 + y^2 + 2z^2, y^2 - 2zx) = 0$

2) Solve the first order quasi linear PDE by the method of characteristics: $x\frac{\partial u}{\partial x} + (u - x - y)\frac{\partial u}{\partial y} = x + 2y$ in $x \geq 0, -\infty < y < \infty$ with $u = 1 + y$ on $x = 1$

3) Reduce the following second order partial differential equations to canonical form and find the general solution: $\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x\,\partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} + 12x$

2018

1) Find the partial differential equation of the family of all tangent planes to the ellipsoid $x^2 + 4y^2 + 4z^2 = 4$ which are not perpendicular to the xy plane.

2) Find the general solution of the partial differential equation:

$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1$.

3) A thin annulus occupies the region. $0 \leq a \leq r \leq b, 0 \leq \theta \leq 2\pi$ The faces are insulated, Along the inner edge the temperature is maintained $0°$, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$ where K is a constant. Determine the temperature distribution in the annulus.

4) Solve $(2D^2 - 5DD' + 2D'^2)Z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$

2017

1) Solve $(D^2 - 2DD' - D'^2)z = e^{x+2y} + x^3 + \sin\ 2x$

2) Let Γ be a closed curve in xy-plane and let S denote the region bounded by the curve Γ. Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y)\ \forall (x, y) \in S$. If f is prescribed at each point (x, y) of S and w is prescribed on the boundary Γ of S then prove that any solution $w = w(x, y)$, satisfying these conditions, is unique.

3) Find a complete integral of the partial differential equation $2(pq + yp + qx) + x^2 + y^2 = 0$.

4) Reduce the equation $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and solve it.

5) Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}; t > 0$, where $c^2 = \frac{T}{m}$, T the constant tension in the string and m is the mass per unit length of the string.

   (i) Find the appropriate solution of the wave equation

   (ii) Find also the solution under the conditions $y(0, t) = 0, y(l, t) = 0$ for all t and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0, y(x, 0) = a\sin\frac{\pi x}{t}, 0 < x < l, a > 0.$

2016

1) Find general equation of surfaces orthogonal to the family of spheres given by

$$x^2 + y^2 + z^2 = cz.$$

2) Find the general integral of the PDE $(y + zx)p - (x + yz)q = x^2 - y^2$

3) Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes though the parabola $4z + x^2 = 0$

4) Solve the particle differential equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y}\frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$

5) Find the temperature $u(x, t)$ in a bar of silver of length 10cm and constant cross section of area $1\ \text{cm}^2$. Let density $p = 10.6\ \text{g/cm}^3$, thermal conductivity $K = 1.04/(\text{cmsec}°\text{C})$ and specific heat $\sigma = 0.056/g°\text{C}$ the bar is perfectly isolated laterally with ends kept at $0°\text{C}$ and initial temperature $f(x) = \sin(0.1\pi x)°C$ note that $u(x, t)$ follows the head equation $u_t = c^2 u_{xx}$ where $c^2 = k/(\rho\sigma)$

2015

1) Solve the partial differential equation: $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$

2) Solve: $(D^2 + DD' - 2D'^2)u = e^{x+y}$,

3) Solve for the general solution $p\cos(x + y) + q\sin(x + y) = z$,

4) Find the solution of the initial-boundary value problem

$$u_t - u_{xx} + u = 0,\ 0 < x < l, t > 0$$
$$u(0,t) = u(l,t) = 0,\ t \geq 0$$
$$u(x,0) = x(l-x),\ 0 < x < l$$

5) Reduce the second-order PDE $x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ into canonical form. Hence, find its general solution

2014

1) Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)z = 24(y-x)$

2) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

3) Find the deflection of a vibrating string (length $= \pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ corresponding to zero initial velocity and initial deflection. $f(x) = k(\sin x - \sin 2x)$

4) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$, given that

   (i) $u(x,0) = 0,\ 0 \leq x \leq 1$

   (ii) $\frac{\partial u}{\partial t}(x,0) = x^2,\ 0 \leq x \leq 1$

   (iii) $u(0,t) = u(1,t) = 0$, for all t

2013

1) From a PDE by eliminating the functions f and g from $z = yf(x) + xg(y)$

2) Reduce the equation $y \frac{\partial^2 z}{\partial x^2} + (x+y) \frac{\partial^2 z}{\partial x \partial y} + x \frac{\partial^2 z}{\partial y^2} = 0$ to its canonical from when $x \neq y$

3) Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x+y)$ where D and D' denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

4) Find the surface which intersects the surfaces of the system $z(x+y) = C(3z+1)$, (C being a constant) orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$

5) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x(l-x)$, find the displacement of the string at any distance x from one end at any time t

2012

1) Solve partial differential equation $(D - 2D')(D - D')^2 z = e^{x+y}$

2) Solve partial differential equation $px + qy = 3z$

3) A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x,t)$ of the vibrating string.

4) The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state.

2011

1) Solve the PDE $(D^2 - D'^2 + D + 3D' - 2)z = e^{(x-y)} - x^2y$

2) Solve the PDE $(x+2z)\frac{\partial z}{\partial x} + (4zx - y)\frac{\partial z}{\partial y} = 2x^2 + y$

3) Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$

4) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \le x \le a, 0 \le y \le b$ satisfying the boundary conditions

$$u(0,y) = 0, u(x,0) = 0, u(x,b) = 0 \frac{\partial u}{\partial x}(a,y) = T\sin^3\frac{\pi y}{a}$$

5) Obtain temperature distribution $y(x,t)$ in a uniform bar of unit length whose one end is kept at 10^0 and the other end is insulated. Also, it is given that $y(x,0) = 1 - x, 0 < x < 1$

2010

1) Solve the PDE $(D^2 - D')(D - 2D')Z = e^{2x+y} + xy$

2) Find the surface satisfying the PDE $(D^2 - 2DD' + D'^2)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$

3) Solve the following partial differential equation

$$zp + yq = x$$
$$x_0(s) = s, y_0(s) = 1, z_0(s) = 2s$$

by the method of characteristics.

4) Reduce the following 2nd order partial differential equation into canonical form and find its general solution. $xu_{xx} + 2x^2u_{xy} - u_x = 0$

5) Solve the following heat equation

$$u_t - u_{xx} = 0, \qquad 0 < x < 2, t > 0$$
$$u(0,t) = u(2,t) = 0 \qquad t > 0$$
$$u(x,0) = x(2-x), \qquad 0 \le x \le 2$$

2009

1) Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$.

2) Form the partial differential equation by elimination the arbitrary function f given by:

$$f(x^2 + y^2, z - xy) = 0$$

3) Find the integral surface of: $x^2p + y^2q + z^2 = 0$ which passes through the curve : $xy = x + y, z = 1$

4) Find the characteristics of: $y^2r - x^2t = 0$ where r and t have their usual meanings.

5) Solve: $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$

6) A tightly stretched string has its ends fixed at $x = 0$ and $x = 1$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t > 0$.

2008

1) Find the general solution of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also find the particular solution which passes through the lines $x = 1, y = 0$

2) Find the general solution of the partial differential equation: $(D^2 + DD' - 6D'^2)z = y\cos x,$

3) Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, x = a, y = 0$ and $y = b$. The edges and $x = 0, x = a$ and $y = 0$ are kept at temperature zero while the edge $y = b$ is kept at $100°$C.

4) Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method.

5) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ canonical form.

2007

1) Form a PDE by eliminating the function f from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

2) Solve $2zx - px^2 - 2qxy + pq = 0$

3) Transform the equation $yz_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution.

4) Solve $u_{xx} + u_{yy} = 0$ in D where $D = \{(x, y): 0 < x < a, 0 < y < b\}$ is a rectangle in a plane with the boundary conditions:

$$u(x, 0) = 0, u(x, b) = 0, 0 \le x \le a$$
$$u(0, y) = g(y), u_x(a, y) = h(y), 0 \le y \le b.$$

5) Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions: $u(0, t) = 0 = u(l, t)$, for all t and $u(x, 0) = f(x)$ for all x in $[0, l]$

2006

1) Solve: $px(z-2y^2)=(z-qy)(z-y^2-2x^3)$

2) Solve: $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial x\partial y^2}=2\sin(3x+2y)$

3) The deflection of vibrating string of length l, is governed by the partial differential equation $u_{tt}=C^2u_{xx}$. The ends of the string are fixed at and $x=0$ and l. The initial velocity is zero.

The initial displacement is given by $u(x,0)=\begin{cases}\frac{x}{l}, & 0<x<\frac{l}{2}\\ \frac{1}{l}(l-x), & \frac{l}{2}<x<l\end{cases}$

Find the deflection of the string at any instant of time.

4) Find the surface passing through the parabolas $z=0, y^2=4ax$ and $z=1, y^2=-4ax$ and satisfying the equation $x\frac{\partial^2 z}{\partial x^2}+2\frac{\partial z}{\partial x}=0$

5) Solve the equation $p^2x+q^2y=z, p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$

2005

1) Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes.

2) Find the particular integral of $x(y-z)p+y(z-x)q=z(x-y)$ which represents a surface passing through $x=y=z$

3) The ends A and B of a rod 20 cm long have the temperature at $30°$C and $80°$C until steady state prevails. The temperatures of ends are changed to $40°$C and $60°$C respectively. Find the temperature distribution in the rod at time t.

4) Obtain the general solution of $(D-3D'-2)^2z=2e^{2x}\sin(y+3x)$

2004

1) Find the integral surface of equation: $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$

2) Find the complete integral of the partial differential equation $(p^2+q^2)x=pz$ and deduce the solution which passes through the curve $x=0, z^2=4y$.

3) Solve the partial differential equation: $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=(y-1)e^x$

4) A uniform string of length l, held tightly between $x=0$ and $x=l$ with no initial displacement, is struck at $x=a, 0<a<l$, with velocity v_0. Find the displacement of the string at any time $t>0$

5) Using Charpit's method, find complete solution of partial differential equation $p^2x+q^2y=z$

2003

1) Find the general solution of $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y+\cos(2x+3y)$

2) Show that the differential equations of all cones which have their vertex at the origin are $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

3) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = xy + e^{x+2y}$

4) Solve the equation $p^2 + q^2 - 2px - 2qy + 2xy = 0$ using Charpit's method. Also find the singular solution of the equation, if it exists.

5) Find the deflection $u(x,t)$ of a vibrating string, stretched between fixed points $(0,0)$ and $(3l,0)$, corresponding to zero initial velocity and following initial deflection:

$$f(x) = \begin{cases} \frac{hx}{l} & \text{when } 0 \le x \le 1 \\ \frac{h(3l-2x)}{l} & \text{when } l \le x \le 2l \\ \frac{h(x-3l)}{l} & \text{when } 2l \le x \le 3l \end{cases}$$

Where h is a constant.

2002

1) Find two complete integrals of the PDE $x^2p^2 + y^2q^2 - 4 = 0$

2) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$

3) Frame the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$

4) Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \frac{x}{2}, y = 0$

5) Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0$

$$u(0,t) = u(l,t) = 0$$
$$u(x,0) = x(l-x),\ 0 \le x \le t$$

2001

1) Find the complete integral partial differential equation $2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2 + y^2)$

2) Find the general integral of $\{my(x+y) - nz^2\}\frac{\partial z}{\partial x} - \{lx(x+y) - nz^2\}\frac{\partial z}{\partial y} = (lx - my)z$

3) Prove that for the equation $z + px + qy - 1 - pqx^2y^2 = 0$ the characteristic strips are given by $x(t) = \frac{1}{B+Ce^{-t}}, y(t) = \frac{1}{A+De^{-t}}, z(t) = E - (AC + BD)e^{-t}$ $p(t) = A(B + Ce^{-t})^2, q(t) = B(A + De^{-t})^2$ where A, B, C, D and E are arbitrary constants. Hence find values of these arbitrary constants if the integral surface passes through the line $z = 0, x = y$

4) Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = C_1y^2$

5) Solve the equation $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = x^2 y^4$ by reducing it to the equation with constant coefficients.