



UPSC CSE Mathematics: Previous Year Questions: Numerical Analysis

2025

- 1) Solve the following system of linear equations by Gauss-Seidel method:

$$\begin{aligned} 10x + 2y + z &= 9 \\ 2x + 20y - 2z &= -44 \\ -2x + 3y + 10z &= 22 \end{aligned}$$

- 2) Convert the number $(3479)_{10}$ into binary system and the number $(7AE \cdot 9F)_{16}$ into decimal system.
- 3) Determine the truth table for the Boolean function $F(x, y, z) = (x + y + z')(x' + y')$
Also derive the full disjunctive normal form of $F(x, y, z)$ from the truth table.
- 4) Simplify the Boolean function $F(x, y, z) = xyz + x'yz + xy'z + xyz'$
and draw the corresponding GATE network.
- 5) Find the unique polynomial of degree 2 or less which fits the following data :

$$\begin{array}{l} x \quad : \quad 0 \quad 1 \quad 3 \\ f(x) \quad : \quad 1 \quad 3 \quad 55 \end{array}$$

Also obtain the bound on the truncation error.

- 6) Find the constant p and error term for the quadrature formula $\int_{x_0}^{x_1} f(x)dx = \frac{h}{2}(f_0 + f_1) + ph^2(f'_0 - f'_1)$ where $x_0 + h = x_1, f_0 = f(x_0), f_1 = f(x_1)$ and prime (') represents derivative with respect to x . Hence deduce the composite rule for integrating $\int_a^b f(x)dx, a = x_0 < x_1 < \dots < x_N = b$

2024

- 1) Using Newton's forward difference formula for interpolation, estimate the value of $f(2 \cdot 5)$ from the following data:

$x :$	1	2	3	4	5	6
$f(x) :$	0	1	8	27	64	125

- 2) Integrate $f(x) = 5x^3 - 3x^2 + 2x + 1$ from $x = -2$ to $x = 4$ using
- (i) Simpson's $\frac{3}{8}$ rule with width $h = 1$, and
- (ii) Trapezoidal rule with width $h = 1$.

- 3) Draw the logical circuit for the Boolean expression $Y = ABC\bar{C} + B\bar{C} + \bar{A}B$. Also, obtain the output Y (truth table) for the three input bit sequences:

$$A = 10001111, B = 00111100, C = 11000100$$

- 4) Solve the following system of linear equations by Gauss-Jordan method

$$2x + 3y - z = 5 \quad 4x + 4y - 3z = 3 \quad 2x - 3y + 2z = 2$$

- 5) Determine the decimal equivalent in sign magnitude form of $(8D)_{16}$ and $(FF)_{16}$.
6) Determine the decimal equivalent of $(9B2.1A)_{16}$.

2023

- 1) Given $\frac{dy}{dx} = \frac{y^2-x}{y^2+x}$ with initial condition $y = 1$ at $x = 0$. Find the value of y for $x = 0.4$ by Euler's method, correct to 4 decimal places, taking step length $h = 0.1$.
2) Evaluate, using the binary arithmetic, the following numbers in their given system :

(i) $(634 \cdot 235)_8 - (132 \cdot 223)_8$

(ii) $(7AB \cdot 432)_{16} - (5CA \cdot D61)_{16}$

- 3) Solve the system of linear equations

$$7x_1 - x_2 + 2x_3 = 11$$

$$2x_1 + 8x_2 - x_3 = 9$$

$$x_1 - 2x_2 + 9x_3 = 7$$

correct up to 4 significant figures by the Gauss-Seidel iterative method. Take initially guessed solution as $x_1 = x_2 = x_3 \equiv 0$.

- 4) Find the conjunctive normal form (CNF) of the following Boolean function:

$$f(x, y, z, t) = x \cdot y \cdot z + \bar{x} \cdot y \cdot (t + \bar{z})$$

- 5) Express the Boolean function $f(x, y, z) = x + \overline{(\bar{x} \cdot \bar{y} + \bar{x} \cdot z)} + z$ in disjunctive normal form (DNF) and construct the truth table for the function.
6) Compute a root of the equation $\log_{10}(2x + 1) - x^2 + 3 = 0$, in the interval $[0, 3]$, by Regula-Falsi method, correct to 6 decimal places.

2022

- 1) Solve, by Gauss elimination method, the system of equations

$$2x + 2y + 4z = 18$$

$$x + 3y + 2z = 13$$

$$3x + y + 3z = 14$$

- 2) Convert the number $(1093 \cdot 21875)_{10}$ into octal and the number $(1693.0628)_{10}$ into hexadecimal systems.
3) Express the Boolean function $F(x, y, z) = xy + x'z$ in a product of maxterms form.

- 4) Find a combinatorial circuit corresponding to the Boolean function

$$f(x, y, z) = [x \cdot (\bar{y} + z)] + y$$

and write the input/output table for the circuit.

- 5) Using Runge-Kutta method of fourth order, solve the differential equation $\frac{dy}{dx} = x + y^2$ with $y(0) = 1$, at $x = 0 \cdot 2$. Use four decimal places for calculation and step length 0-1.
- 6) The velocity of a train which starts from rest is given by the following table, the time being reckoned in minutes from the start and the velocity in km/ hour :

t (minutes)	2	4	6	8	10	12	14	16	18	20
v (km/hour)	16	28.8	40	46.4	51.2	32	17.6	8	3.2	0

Using Simpson's $\frac{1}{3}$ rd rule, estimate approximately in km the total distance run in 20 minutes.

2021

- 1) Find a positive root of the equation $3x = 1 + \cos x$ by a numerical technique using initial values $0, \frac{\pi}{2}$; and further improve the result using Newton-Raphson method correct to 8 significant figures.
- 2) Convert $(3798 \cdot 3875)_{10}$ into octal and hexadecimal equivalents.
- 3) Obtain the principal conjunctive normal form of $(7P \rightarrow R) \wedge (Q \rightleftharpoons P)$.
- 4) Solve the system of equations

$$\begin{aligned} 3x_1 + 9x_2 - 2x_3 &= 11 \\ 4x_1 + 2x_2 + 13x_3 &= 24 \\ 4x_1 - 2x_2 + x_3 &= -8 \end{aligned}$$

correct up to 4 significant figures by using Gauss-Seidel method after verifying whether the method is applicable in your transformed form of the system.

- 5) Derive Newton's backward difference interpolation formula and also do error analysis.
- 6) Obtain the Boolean function $F(x, y, z)$ based on the table given below. Then simplify $F(x, y, z)$ and draw the corresponding GATE network:

x	y	z	F(x, y, z)
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

2020

- 1) Show that the equation $f(x) \equiv \cos \frac{\pi(x+1)}{8} + 0.148x - 0.9062 = 0$ has one root in the interval $(-1,0)$ and one in $(0,1)$. Calculate the negative root correct to four decimal places using Newton-Raphson Method.
- 2) Let $g(w, x, y, z) = (w + x + y)(x + \bar{y} + z)(w + \bar{y})$ be a Boolean function. Obtain the conjunctive normal form for $g(w, x, y, z)$. Also express $g(w, x, y, z)$ as product of maxterms.
- 3) For the solution of system of equations:

$$\begin{aligned} 4x + y + 2z &= 4 \\ 3x + 5y + z &= 7 \\ x + y + 3z &= 3 \end{aligned}$$

Set up the Gauss-Seidel iterative scheme and iterate three times starting with initial vector $X^{(0)} = 0$. Also find the exact solutions and compare with the iterated solutions.

- 4) Find a quadrature formula $\int_0^1 f(x) \frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$ which is exact for polynomials of highest possible degree. Then use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ (correct up to three decimal places).
- 5) Write the three-point Lagrangian interpolating polynomial relative to the points $x_0, x_0 + \varepsilon$ and x_1 . Then by taking the limit $\varepsilon \rightarrow 0$, establish the relation

$$f(x) = \frac{(x_1 - x)(x + x_1 - 2x_0)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x_1 - x)}{(x_1 - x_0)} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)} f(x_1) + E(x)$$

where $E(x) = \frac{1}{6}(x_1 - x_0)^2(x - x_1)f'''(\xi)$ is the error function and $\min(x_0, x_0 + \varepsilon, x_1) < \xi < \max(x_0, x_0 + \varepsilon, x_1)$.

2019

- 1) Apply Newton-Raphson method, to find real root of transcendental equation, $x \log_{10} x = 1.2$ correct to three decimal places.
- 2) Using Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$. Use four decimal places for calculation and step length 0.2
- 3) Draw a flow chart and write a basic algorithm for (in FORTRAN/C/C++) for evaluating $y = \int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule
- 4) Find the equivalent numbers given in a specified number to the system mentioned against them:
 - (i) Integer 524 in binary system.
 - (ii) 101010110101.101101011 to octal system.
 - (iii) decimal number 5280 to hexadecimal system.

- (iv) Find the unknown number $(1101.101)_8 \rightarrow (?)_{10}$.
- 5) Apply Gauss-Seidel iteration method to solve the following system of equations:
 $2x + y - 2z = 17$ $3x + 20y - z = 182$ $x - 3y + 20z = 25$, correct to three decimal places.
- 6) Given the Boolean expression. $X = AB + ABC + A\bar{B}\bar{C} + \bar{A}\bar{C}$
- Draw the logical diagram for the expression.
 - Minimize the expression.
 - Draw the logical diagram for the reduced expression.

2018

- Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1, u_2 = 9, u_3 = 25, u_4 = 55$ and $u_5 = 105$.
- Starting from rest in the beginning, the speed (in km/h) of a train at different times (in minutes) is given by the below table:

समय (मिनट) Time (Minutes)	2	4	6	8	10	12	14	16	18	20
रफ़्तार (किमी/घं) Speed (Km/h)	10	18	25	29	32	20	11	5	2	8.5

Using Simpsons' $\frac{1}{3}rd$ rule, Find the approximate distance travelled (in km) in 20 minutes from the beginning.

- Write down the basic algorithm for solving the equation $xe^x - 1 = 0$ by bisection method, correct to 4 decimal places.
- Find the equivalent of numbers given in a specified number system to the system mentioned against them.
 - $(111011 \cdot 101)_2$ to decimal system
 - $(1000111110000 \cdot 00101100)_2$ to hexadecimal system
 - $(C4 F2)_{16}$ to decimal system
 - $(418)_{10}$ to binary system
- Simplify the Boolean expression: $(a + b) \cdot (\bar{b} + c) + b \cdot (\bar{a} + \bar{c})$ By using the laws of Boolean algebra. From its truth table write it in min-terms normal form.
- Find the values of the constants a, b, c such that the quadrature formula $\int_0^h f(x)dx = h \left[af(0) + bf\left(\frac{h}{3}\right) + cf(h) \right]$ is exact for polynomials of as high degree as possible, and hence find the order of the truncation error.

2017

- 1) Explain the main steps of the Gauss-Jordan method and apply this method to find the inverse of the matrix $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$
- 2) Write the Boolean expression $z(y + z)(x + y + z)$ in the simplest form using Boolean postulate rules. Mention the rules used during simplification. Verify your result by constructing the truth table for the given expression and for its simplest form.
- 3) For given equidistant values u_{-1}, u_0, u_1 and u_2 a values are interpolated by Lagrange's formula. Show that it may be written in the form $u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2u_0$, where $x + y = 1$.
- 4) Derive the formula $\int_a^b y dx = \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-1}) + 2(y_3 + y_6 + y_{n-3})]$. Is there any restriction on n ? State that condition. What is the error bounded in the case of Simpson's $\frac{3}{8}$ rule?
- 5) Write an algorithm in the form of a flow chart for Newton-Raphson method. Describe the cases of failure of this method.

2016

- 1) Convert the following decimal numbers to univalent binary and hexadecimal numbers:
[i] 4096 [ii] 0.4375 [iii] 2048.0625
- 2) Let $f(x) = e^{2x} \cos 3x$ for $x \in [0,1]$. Estimate the value of $f(0.5)$ Using Lagrange interpolating polynomial of degree 3 over the nodes $x = 0, x = 0.3, x = 0.6$ and $x = 1$. Also compute the error bound over the interval $[0,1]$ and the actual error $E(0.5)$
- 3) For an integral $\int_{-1}^1 f(x) dx$ show that the two-point Gauss quadrature rule is given by $\int_{-1}^1 f(x) dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$ using this rule estimate $\int_2^4 2xe^x dx$
- 4) Let A, B, C be Boolean variable denote complement $\bar{A}, \bar{A} + B$ of is an expression for $A \text{ OR } B$ and $B \cdot A$ is an expression for $A \text{ AND } B$. Then simplify the following expression and draw a block diagram of the simplified expression using AND and OR gates.

$$A \cdot (A + BC) \cdot (\bar{A} + B + C) \cdot (A + \bar{B} + C) \cdot (A + B + \bar{C}).$$

2015

- 1) Find the principal [or canonical] disjunctive normal form in three variables p, q, r for the Boolean expression $((p \wedge q) \rightarrow r) \vee ((p \wedge q) \rightarrow \neg r)$. Is the given Boolean expression a contradiction or a tautology?

- 2) Find the Lagrange interpolating polynomial that fits the following data:

x	:	-1	2	3	4
$f(x)$:	-1	11	31	69

Find $f(1.5)$

- 3) Solve the initial value problem $\frac{dy}{dx} = x(y - x), y(2) = 3$ in the interval $[2, 2.4]$ using the RungeKutta fourth-order method with step size $h = 0.2$
- 4) Find the solution of the system

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

using Gauss-Seidel method (make four iterations)

2014

- 1) Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct up to four decimal places.
- 2) Use five subintervals to integrate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule.
- 3) Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z = xy + uv$
- 4) Solve the system of equations using Gauss-Seidel iteration method [perform three iterations]

$$\begin{aligned} 2x_1 - x_2 &= 7 \\ -x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 2x_3 &= 1 \end{aligned}$$

- 5) Use Runge-Kutta formula of fourth order to find the value of y at $x = 0.8$, where

$$\frac{dy}{dx} = \sqrt{x + y}, y(0.4) = 0.41. \text{ Take the step length } h = 0.2$$

- 6) Draw a flowchart for Simpson's one-third rule.
- 7) For any Boolean variables x and y , show that $x + xy = x$.

2013

- 1) In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No. of Students	31	42	51	35	31

Using Newton's forward interpolation formula, find the number of students whose marks lie between 45 and 50.

- 2) Develop an algorithm for Newton-Raphson method to solve $f(x) = 0$ starting with initial iterate x_0 , n be the number of iterations allowed, epsilon be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$
- 3) Use Euler's method with step size $h = 0.15$ to compute the approximate value of $y(0.6)$, correct up to five decimal places from the initial value problem. $y' = x(y + x) - 1, y(0) = 2$
- 4) The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/ hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule.

2012

- 1) Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$ correct to four decimal places
- 2) Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval $[a, b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method.
- 3) Solve the following system of simultaneous equations, using Gauss-Seidel iterative method:

$$3x + 20y - z = -18 \quad 20x + y - 2z = 17 \quad 2x - 3y + 20z = 25$$

- 4) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data:

x :	0.1	0.2	0.3	0.4
y :	0.9975	0.9900	0.9776	0.9604

- 5) In a certain examination, a candidate has to appear for one major and two minor subjects. The rules for declaration of results are : marks for major are denoted by M_1 and for minors by M_2 and M_3 . If the candidate obtains 75% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class with distinction. If the candidate obtains 60% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class. If the candidate obtains 50% or above

in major, 40% or above in each of the two minors and an average of 50% or above in all the three subjects put together, the candidate is declared to have passed the examination in second class. All those candidates, who have obtained 50% and above in major and 40% or above in minor, are declared to have passed the examination. If the candidate obtains less than 50% in major or less than 40% in any one of the two minors, the candidate is declared to have failed in the examinations. Draw a flow chart to declare the results for the above.

2011

- 1) Calculate $\int_2^{10} \frac{dx}{1+x}$ [up to 3 places of decimal] by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}$ rd rule.
- 2) Compute $(3205)_{10}$ to the base 8.
- 3) Let A be an arbitrary but fixed Boolean algebra with operations \wedge, \vee and $'$ and the zero and the unit element denoted by 0 and 1 respectively. Let $x, y, z \dots$ be elements of A . If $x, y \in A$ be such that $x \wedge y = 0$ and $x \vee y = 1$ then prove that $y = x'$
- 4) A solid of revolution is formed by rotating about the x -axis, the area between the x -axis, the line $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates:

x	.00	.25	.50	.75	1
y	1	.9896	.9589	.9089	.8415

Find the volume of the solid.

- 5) Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit:

x	y	z	f(x, y, z)
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

- 6) Draw a flow chart for Lagrange's interpolation formula.

2010

- 1) Find the positive root of the equation $10xe^{-x^2} - 1 = 0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations.

- 2) Suppose a computer spends 60 per cent of its time handling a particular type of computation when running a given program and its manufacturers make a change that improves its performance on that type of computation by a factor of 10. If the program takes 100 sec to execute, what will its execution time be after the change?
- 3) If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$.
- 4) Given the system of equations

$$\begin{aligned} 2x + 3y &= 1 \\ 2x + 4y + z &= 2 \\ 2y + 6z + Aw &= 4 \\ 4z + Bw &= C \end{aligned}$$

State the solvability and uniqueness conditions for the system. Give the solution when it exists.

- 5) Find the value of the integral $\int_1^5 \log_{10} x dx$ by using Simpson's $\frac{1}{3}$ rd rule correct up to 4 decimal places. Take 8 subintervals in your computation.
- 6) Find the hexadecimal equivalent of the decimal number $(587632)_{10}$
- 7) For the given set of data points $(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_n, f(x_n))$. Write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula
- 8) Using Boolean algebra, simplify the following expressions
- [a] $a + a'b + a'b'c + a'b'c'd + \dots$
- [b] $x'y'z + yz + xz$ where x' represents the complement of x

2009

- 1) The equation $x^2 + ax + b = 0$ has two real roots α and β . Show that the iterative method given by: $x_{k+1} = -\frac{(ax_k+b)}{x_k}, k = 0,1,2 \dots$ is convergent near $x = \alpha$, if $|\alpha| > |\beta|$
- 2) Find the values of two valued Boolean variables A, B, C, D by solving the following simultaneous equations:
- $$\begin{aligned} \bar{A} + AB &= 0 \\ AB + AC & \\ AB + A\bar{C} + CD &= \bar{C}D \end{aligned}$$
- where \bar{x} represents the complement of x
- 3) Realize the following expressions by using NAND gates only: $g = (\bar{a} + \bar{b} + c)\bar{d}(\bar{a} + e)f$ where \bar{x} represents the complement of x .
- 4) Find the decimal equivalent of $(357.32)_8$
- 5) Develop an algorithm for Regula-Falsi method to find a root of $f(x) = 0$ starting with two initial iterates x_0 and x_1 to the root such that $\text{sign}(f(x_0)) \neq \text{sign}(f(x_1))$. Take n as the maximum number of iterations allowed and epsilon be the prescribed error.

- 6) Using Lagrange interpolation formula, calculate the value of $f(3)$ from the following table of values of x and $f(x)$:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

- 7) Find the value of $y(1.2)$ using Runge-Kutta fourth order method with step size $h = 0.2$ from the initial value problem: $y' = xy, y(1) = 2$

2008

- 1) Find the smallest positive root of equation $xe^x - \cos x = 0$ using Regula-Falsi method. Do three iterations.

- 2) State the principle of duality

(i) in Boolean algebra and give the dual of the Boolean expressions

$$(X + Y) \cdot (\bar{X} \cdot \bar{Z}) \cdot (Y + Z) \text{ and } X\bar{X} = 0$$

(ii) Represent $(\bar{A} + \bar{B} + \bar{C})(A + \bar{B} + C)(A + B + \bar{C})$ in NOR to NOR logic network.

- 3) The following values of the function $f(x) = \sin x + \cos x$ are given:

x	10^0	20^0	30^0
$f(x)$	1.1585	1.2817	1.3360

Construct the quadratic interpolating polynomial that fits the data. Hence calculate $f\left(\frac{\pi}{12}\right)$.

Compare with exact value.

- 4) Apply Gauss-Seidel method to calculate x, y, z from the system:

$$-x - y + 6z = 42$$

$$6x - y - z = 11.33$$

$$-x + 6y - z = 32$$

with initial values $(4.67, 7.62, 9.05)$. Carry out computations for two iterations

- 5) Draw a flow chart for solving equation $F(x) = 0$ correct to five decimal places by Newton-Raphson method

2007

- 1) Use the method of false position to find a real root of $x^3 - 5x - 7 = 0$ lying between 2 and 3 and correct to 3 places of decimals.

- 2) Convert:

(i) 46655 given to be in the decimal system into one in base 6.

(ii) $(11110.01)_2$ into a number in the decimal system.

- 3) Find from the following table, the area bounded by the x -axis and the curve $y = f(x)$ between $x = 5.34$ and $x = 5.40$ using the trapezoidal rule:

x	5.34	5.35	5.36	5.37	5.38	5.39	5.40
$f(x)$	1.82	1.85	1.86	1.90	1.95	1.97	2.00

- 4) Apply the second order Runge-Kutta method to find an approximate value of y at $x = 0.2$ taking $h = 0.1$, given that y satisfies the differential equation and initial condition $y' = x + y, y(0) = 1$

2006

- 1) Evaluate $I = \int_0^1 e^{-x^2} dx$ by the Simpson's rule

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2)] + 4f(x_3) + \dots + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})]$$
 with $2n = 10, \Delta x = 0.1, x_0 = 0, x_1 = 0.1, \dots, x_{10} = 1.0$
- 2) [i] Given the number 59.625 in decimal system. Write its binary equivalent.
 [ii] Given the number 3898 in decimal system. Write its equivalent in system base 8.
- 3) If Q is a polynomial with simple roots $\alpha_1, \alpha_2, \dots, \alpha_n$ and if P is a polynomial of degree $< n$, show that $\frac{P(x)}{Q(x)} = \sum_{k=1}^n \frac{P(\alpha_k)}{Q'(\alpha_k)(x-\alpha_k)}$. Hence prove that there exists a unique polynomial of degree with given values c_k at the point $\alpha_k, k = 1, 2, \dots, n$.
- 4) Draw a flowchart and algorithm for solving the following system of 3 linear equations in 3 unknowns x_1, x_2 & $x_3: C * X = D$ with $C = (c_{ij})_{i,j=1}^3, X = (x_j)_{j=1}^3, D = (d_i)_{i=1}^3$

2005

- 1) Use appropriate quadrature formulae out of the Trapezoidal and Simpson's rules to numerically integrate $\int_0^1 \frac{dx}{1+x^2}$ with $h = 0.2$. Hence obtain an approximate value of π . Justify the use of particular quadrature formula.
- 2) Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011.10)_2$
- 3) Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1, P(3) = 27, P(4) = 64$. Using the Lagrange's interpolation formula and the Newton's divided difference formula, evaluate $P(1.5)$
- 4) Draw a flow chart and also write algorithm to find one real root of the nonlinear equation $x = \phi(x)$ by the fixed-point iteration method. Illustrate it to find one real root, correct up to four places of decimals, of $x^3 - 2x - 5 = 0$

2004

- 1) The velocity of a particle at distance from a pint on its path is given by the following table:

$S(\text{meters})$	0	10	20	30	40	50	60
$V(\text{m/sec})$	47	58	64	65	61	52	38

Estimate the time taken to travel the first 60 meters using Simpson's $\frac{1}{3}$ rd rule. Compare the result with Simpson's $\frac{3}{8}$ th rule.

- 2) [i] If $(ABCD)_{16} = (x)_2 = (y)_8 = (z)_{10}$ then find $x, y \& z$
 [ii] In a 4-bit representation, what is the value of 1111 in signed integer form, unsigned integer form, signed 1's complement form and signed 2's complement form?
- 3) How many positive and negative roots of the equation $e^x - 5\sin x = 0$ exist? Find the smallest positive root correct to 3 decimals, using Newton-Raphson method.
- 4) Using Gauss-Seidel iterative method, find the solution of the following system:

$$\begin{aligned} 4x - y + 8z &= 26 \\ 5x + 2y - z &= 6 \text{ up to three iterations.} \\ x - 10y + 2z &= -13 \end{aligned}$$

2003

- 1) Evaluate $\int_0^1 e^{-x^2} dx$ by employing three points Gaussian quadrature formula, finding the required weights and residues. Use five decimal places for computation.
- 2) [i] Convert the following binary number into octal and hexa decimal system:
101110010.10010
- 3) [ii] Find the multiplication of the following binary numbers: 11001.1 and 101.1
- 4) Find the positive root of the equation $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$ using Newton-Raphson method correct to four decimal places. Also show that the following scheme has error of second order:

$$x_{n+1} = \frac{1}{2}x_n \left(1 + \frac{a}{x_n^2}\right)$$
- 5) Draw flow chart and algorithm for Simpson's $\frac{1}{3}$ rd rule for integration $\int_a^b \frac{1}{1+x^2} dx$ correct to 10^{-6}

2002

- 1) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position.
- 2) Convert $(100.85)_{10}$ into its binary equivalent.
- 3) Multiply the binary numbers $(1111.01)_2$ and $(1101.11)_2$ and check with its decimal equivalent

- 4) Find the cubic polynomial which takes the following values: $y(0) = 1, y(1) = 0, y(2) = 1$ & $y(3) = 10$. Hence, or otherwise, obtain $y(4)$
- 5) Given: $\frac{dy}{dx} = y - x$ where $y(0) = 2$, using the Runge-Kutta fourth order method, find $y(0.1)$ and $y(0.2)$. Compare the approximate solution with its exact solution. ($e^{0.1} = 1.10517, e^{0.2} = 1.2214$).
- 6) Draw a flow chart to examine whether a given number is a prime.

2001

- 1) Show that the truncation error associated with linear interpolation of $f(x)$, using ordinates at x_0 and x_1 with $x_0 \leq x \leq x_1$ is not larger in magnitude than $\frac{1}{8}M_2(x_1 - x_0)^2$ where $M_2 = \max|f''(x)|$ in $x_0 \leq x \leq x_1$. Hence show that if $f(x) = \frac{2}{\sqrt{\pi}} \int_0^{\pi} e^{-t^2} dt$, the truncation error corresponding to linear interpolation of $f(x)$ in $x_0 \leq x \leq x_1$ cannot exceed $\frac{(x_1 - x_0)^2}{2\sqrt{2}\pi e}$.
- 2) Given $A \cdot B' + A' \cdot B = C$ show that $A \cdot C' + A' \cdot C = B$
- 3) Express the area of the triangle having sides of lengths $6\sqrt{2}, 12, 6\sqrt{2}$ units in binary number system.
- 4) Using Gauss Seidel iterative method and the starting solution $x_1 = x_2 = x_3 = 0$, determine the solution of the following system of equations in two iterations

$$\begin{aligned} 10x_1 - x_2 - x_3 &= 8 \\ x_1 + 10x_2 + x_3 &= 12. \\ x_1 - x_2 + 10x_3 &= 10 \end{aligned}$$

Compare the approximate solution with the exact solution

- 5) Find the values of the two-valued variables A, B, C & D by solving the set of simultaneous

$$A' + A \cdot B = 0$$

equations $A \cdot B = A \cdot C$

$$A \cdot B + A \cdot C' + C \cdot D = C' \cdot D$$