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UPSC CSE Mathematics: Previous Year Questions: Mechanics

2025

1) A particle of mass m moves in a force field of potential $V(r) = -\frac{k\cos\theta}{r^2}$, k is constant

Find the Hamiltonian and the Hamilton's equations in spherical polar coordinates (r, θ, ϕ).

2) Consider the Lagrangian $L = m\dot{x}\dot{y} - m\omega_0^2 xy$ where m and ω_0 are constants. Find the Hamiltonian and Hamilton's equations of motion. Identify the system.

3) Calculate the moment of inertia of a uniform solid cylinder of mass M, radius R and length L with respect to a set of axes passing through the centre of the cylinder, where z-axis is the axis of the cylinder and ρ is the constant density at any point of the cylinder. Also find $\frac{L}{R}$ for which the moment of inertia about x - or y-axis will be minimum for a given mass of the cylinder.

4) A bead of mass m slides on a frictionless wire in the shape of a cycloid given by $x = a(\theta - \sin\ \theta), y = a(1 + \cos\ \theta), (0 \le \theta \le 2\pi)$. Find the Lagrangian function. Hence show that the equation of motion can be written as $\frac{d^2u}{dt^2} + \frac{g}{4a}u = 0$ where $u = \cos\left(\frac{\theta}{2}\right)$.

2024

1) Find the moment of inertia of a quadrant of an elliptic disk $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, of mass M about the line passing through its centre and perpendicular to its plane. Given that the density at any point is proportional to xy

2) A rough uniform board of mass m and length 2a rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance covered by the board during this time.

2023

1) A mechanical system with 2 degrees of freedom has the Lagrangian

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}m(w_1^2x^2 + w_2^2y^2) + kxy$$

where m, w_1, w_2, k are constants. Find the parameter θ so that under the transformation $x = q_1\cos\ \theta - q_2\sin\ \theta,\ y = q_1\sin\ \theta + q_2\cos\ \theta$

the Lagrangian in terms of q_1, q_2 will not contain the product term q_1q_2. Find the Lagrange's equations w.r.t. q_1 and q_2 independent of parameter θ.

2) A perfectly rough ball is at rest within a hollow cylindrical roller. The roller is drawn along a level path with uniform velocity V. Let a and b be the radii of the ball and the roller respectively. If $V^2 > \frac{27}{7}g(b-a)$, then show that the ball will roll completely round the inside of the rotter.

3) A planet of mass m is revolving around the sun of mass M. The kinetic energy T and the potential energy V of the planet are given by $T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$ and $V = GMm\left(\frac{1}{2a} - \frac{1}{r}\right)$, where (r, θ) are the polar coordinates of the planet at time t, G is the gravitational constant and $2a$ is the major axis of the ellipse (the path of the planet). Find the Hamiltonian and the Hamilton equations of the planet's motion.

2022

1) A particle at a distance r from the centre of force moves under the influence of the central force $F = -\frac{k}{r^2}$, where k is a constant. Obtain the Lagrangian and derive the equations of motion.

2) Find the moment of inertia of a right circular solid cone about one of its slant sides (generator) in terms of its mass M, height h and the radius of base as a.

2021

1) A particle is constrained to move along a circle lying in the vertical xy-plane. With the help of the D'Alembert's principle, show that its equation of motion is $\ddot{x}y - \ddot{y}x - gx = 0$, where g is the acceleration due to gravity.

2) Obtain the Lagrangian equation for the motion of a system of two particles of unequal masses connected by an inextensible string passing over a small smooth pulley

2020

1) Prove that the moment of inertia of a triangular lamina ABC about any axis through in A its plane is $\frac{M}{6}(\beta^2 + \beta\gamma + \gamma^2)$ where M is the mass of the lamina and β, γ are respectively the length of perpendiculars from B and C on the axis.

2) By writing down the Hamiltonian, find the equations of motion of a particle of mass m constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$ is a constant. The particle is subject to a force directed towards the origin and proportional to the distance r of the particle from the origin given by $\vec{F} = -k\vec{r}, k$ is a constant

2019

1) A uniform rod OA of length $2a$ free to turn about it end O revolves with angular velocity about the vertical OZ through O and is inclined at a constant angle α to OZ; find value of α

2) A circular cylinder of radius and radius a of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane thorough axes move in a circular pendulum of length $(b-a)\left(1+\frac{k^2}{a^2}\right)$.

3) Using Hamilton's equation find the acceleration for a sphere rolling down a rough inclined plane, if x be the distance of the point of contact of the sphere from a fixed point on the plane.

2018

1) Suppose the Lagrangian of a mechanical system is given by

$L = \frac{1}{2}m(ax^2 + 2bxy + cy^2) - \frac{1}{2}k(ax^2 + 2bxy + cy^2)$, Where $a, b, c, m(>0), k(>0)$ are constants and $b^2 \neq ac$. write down the Lagrangian equations of motion and identify the system.

2) The Hamiltonian of a mechanical system is given by $H = p_1q_1 - aq_1^2 + bq_2^2 - p_2q_2$ Where a, b are the constants. Solve the Hamiltonian equations and show that $\frac{p_2 - bq_2}{q_1} =$ constant.

2017

1) Show that moment if inertia of an elliptic area of mass M and semi-axis a and b about a semi-diameter of length r is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. Further, prove that the moment of inertia about a tangent is $\frac{5M}{4}p^2$ where p is the perpendicular distance from the centre of the ellipse to the tangent.

2) Two uniform rods AB, AC each of mass m and length $2a$, are smoothly hinged together at A and move on a horizontal plane. At time t, the mass centre of the road is at the point (ξ, η) referred to fixed perpendicular axes Ox, Oy in the plane, and the rods make angles $\theta \pm \phi$ with Ox. Prove that the kinetic energy of the system is $m\left[\xi^2 + \eta^2 + \left(\frac{1}{3} + \sin^2 \phi\right)a^2\theta^2 + \left(\frac{1}{3} + \cos^2 \phi\right)a^2\phi^2\right]$. Also derive Lagrange's equation of motion for the system if an external force with components $[X, Y]$ along the axes acts at A.

2016

1) Consider single free particle of mass m, moving in space under no forces. If the particle starts form the origin $t = 0$ at and reaches the position (x, y, z) at time τ, find the Hamilton's characteristic function S as a function of x, y, z, τ.

2) A hoop with radius r is rolling, without slipping, down an inclined plane of length l and with angle of inclination ϕ. Assign appropriate generalized coordinate to the system. Determine the constraints, if any. Write down the Lagrangian equation for the system. Hence or otherwise determine the velocity of the hoop at the bottom of the inclined plane.

2015

1) Calculate the moment of inertia of a solid uniform hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$ with mass m about the OZ-axis.

2) Solve the plane pendulum problem using the Hamiltonian approach and show that H is a constant of motion.

3) A Hamiltonian of a system with one degree of freedom has form

$H = \frac{p^2}{2\alpha} - bape^{-\alpha t} + \frac{b\alpha}{2} q^2 e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{k}{2} q^2$ where α, b, k are constant, q is the generalized coordinate and p is the corresponding generalized momentum.

(i) Find a Lagrangian corresponding too this Hamiltonian.

(ii) Find an equivalent Lagrangian that is not explicitly dependent on time.

2014

1) Find the equation of motion of a compound pendulum using Hamilton's equations.

2013

1) For solid sphere A, B, C and D, each of mass m and radius a, are placed with their centers on the four corners of a square of side b. Calculate the moment of inertia of the system about a diagonal of the square.

2) Two equal rod AB and BC, each of length l, smoothly jointed at B, are suspended from A and oscillate in a vertical plane through A. Show that that the periods of normal oscillation are $\frac{2\pi}{n}$ where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$

2012

1) Obtain the equations governing the motion of a spherical pendulum.

2) A pendulum consists of a rod of length $2a$ and mass m; to one end of which a spherical bob of radius $\frac{a}{3}$ and mass 15 m is attached. Find the moment of inertia of the pendulum:

   a) About an axis through the other end of the rod and at right angle to the rod.

   b) About a parallel axis through the centre of mass of the pendulum. [Given: the centre of mass of the pendulum is $\frac{a}{12}$ above the centre of the sphere].

2011

1) Let a be the radius of the base of a right circular cone of height h and mass M. Find the moment of inertia of that right circular cone about a line through the vertex perpendicular to the axis.

2) The ends of a heavy rod of length $2a$ are rigidly attached to two light rings which can respectively slide on the thin smooth fixed horizontal and vertical wires O_x and O_y. The rod starts at an angle α to the horizon with an angular velocity $\sqrt{[3g(1-\sin\ \alpha)/2a]}$ and moved downwards. Show that it will strike the horizontal wire at the end of time $-2\sqrt{a/(3g)}\log\left[\tan\left(\frac{\pi}{8}-\frac{\alpha}{4}\right)\cot\frac{\pi}{8}\right]$.

2010

1) A uniform lamina is bounded by a parabolic arc of latus rectum $4a$ and a double ordinate at a distance b from the vertex. If $b=\frac{a}{3}(7+4\sqrt{7})$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there.

2) A sphere of radius a land mass m rolles down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equation.

2009

1) The flat surface of a hemisphere of radius r is cemented to one flat surface of a cylinder of the same radius and of the same material. If the length of the cylinder be l and the total mass be m, show that the moment of inertia of the combination about the axis of the cylinder is given by: $mr^2\frac{\left(\frac{l}{2}+\frac{4}{15}r\right)}{\left(l+\frac{2r}{3}\right)}$.

2) A perfectly rough sphere of mass m and radius b, rests on the lowest point of a fixed spherical cavity a radius a. To the highest point of the movable sphere is attached a particle of mass m ' and the system is disturbed. Show that the oscillations are the same as of a simple pendulum of length $(a-b)\frac{4m'+\frac{7}{5}m}{m+m'\left(2-\frac{a}{b}\right)}$.

2008

1) A circular board is placed on a smooth horizontal plane and a boy runs round the edge of it at a uniform rate. What is the motion of the centre of the board? Explain. What happens if the mass of the board and boy are equal.

2) A uniform road of mass $3m$ and length $2l$ has its middle point fixed and a mass m is attached to one of its extremity. The rod, when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity $\sqrt{\frac{2g}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}(\sqrt{2}-1)$.

2007

1) Consider a system with two degree of freedom for which $V=q_1^2+3q_1q_2+4q_2^2$. Find the equilibrium position and determine if the equilibrium is stable.

2) A point mass m is placed on a frictionless plane that is tangent to the Earth's surface. Determine Hamilton's equations by taking x or θ as the generalized coordinate.

2006

1) Given points $\mathrm{A}(0,0)$ and $\mathrm{B}(x_0,y_0)$ not in the same vertical, it is required to find a curve in the $x-y$ plane joining A to B so that a particle starting from rest will traverse from A to B along this curve without friction in the shortest possible time. If $y=y(x)$ is the required curve find the function $f(x,y,z)$ such that equation of motion can be written as $\frac{dx}{dt}=f(x,y(x),y'(x))$.

2) A particle of mass m is constrained to move on the surface of a cylinder. The particle is subject to a force directed towards the origin and proportional to the distance to of particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion.

2005

1) A rectangular plate swings in a vertical plane about one of its corners. If its period is one second, find the length of its diagonal.

2) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time. $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ Where a is the length of the plank?

2004

1) A particle of mass m moves under the influence of gravity on the inner surface of the paraboloid of revolution $x^2+y^2=az$ which is assumed frictionless. Obtain the equation of motion show that it will describe a horizontal circle in the plane $z=h$, provided that it is given an angular velocity whose magnitude is $\omega=\sqrt{\frac{2g}{a}}$.

2) Derive the Hamilton equations of motion from the principle of least action and obtain the same for a particle of mass m moving in a force field of potential V. Write these equations in spherical coordinates (r,θ,ϕ).

2003

1) A solid body of density ρ is in the shape of the solid formed by the revolution of the Cardioid $r=a(1+\cos\theta)$ about the initial line Show that the moment of inertia about the straight line through the pole and perpendicular to the initial line is $\frac{352}{105}\pi\rho a^2$.

2) A fine circular tube, radius c, lies on a smooth horizontal plane and contains two equal particles connected by an elastic string in the tube, the natural length of which is equal to half the circumference. The particles are in contact and fastened together, the string being stretched the tube. If the particles become disunited, prove that the velocity to the tube when the string has regained its natural length is $\left\{\frac{2\pi\lambda mc}{M(M+2m)}\right\}^{\frac{1}{2}}$ where M,m the masses of the tube and each particle respectively, and is λ the modulus of elasticity.

2002

1) Find the moment of inertia of a circular wire about (i) a diameter and (ii) a line through the centre and perpendicular to its plane.

2) A thin circular disc of mass M and radius a can turn freely about a thin axis OA which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity w about its end A. Show that the

inclination ϕ to the vertical of the radius of the disc through O is $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$ unless $w^2 < \frac{g}{a}$ and then θ is zero.

2001

1) Determine the moment of inertia of a uniform hemisphere about its axis of symmetry and about an axis perpendicular to the axis of symmetry and though centre of the base.

2) Find the equation of motion for a particular of mass m which is constrained to move on the surface of a cone of semi-vertical angle α and which is subjected to a gravitational force.