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UPSC CSE Mathematics: Previous Year Questions: Fluid Dynamics

2025

1) A source and a sink of equal strength are placed at points $\left(\pm\frac{a}{2},0\right)$ within a fixed circular boundary $x^2+y^2=a^2$. Show that the streamlines are given by

$$\left(r^2-\frac{a^2}{4}\right)(r^2-4a^2)-4a^2y^2=ky(r^2-a^2)$$

where k is a constant and $r^2=x^2+y^2$.

2) Show that for an incompressible steady flow with constant viscosity, the velocity components

$$u(y)=\left(\frac{U}{h}\right)y-\frac{hy}{2\mu}\frac{dp}{dx}\left(1-\frac{y}{h}\right)$$

$v=0=w$, with $p=p(x)$, satisfy the equation of motion in the absence of body force. Given that U, h and $\frac{dp}{dx}$ are constants.

2024

1) Suppose an infinite liquid contains two parallel, equal and opposite rectilinear vortices at a distance 2a. Show that the streamlines relative to the vortex are given by the equation

$$\log\frac{x^2+(y-a)^2}{x^2+(y+a)^2}+\frac{y}{a}=C$$

where C is a constant, the origin is the middle point of the join, and the line joining the vortices is the axis of y.

2) The velocity potential ϕ of a flow is given by $\phi=\frac{1}{2}(x^2+y^2-2z^2)$. Determine the streamlines.

3) Let the velocity field $u(x,y)=\frac{B(x^2-y^2)}{(x^2+y^2)^2}$, $v(x,y)=\frac{2Bxy}{(x^2+y^2)^2}$, $w(x,y)=0$ satisfy the equations of motion for inviscid incompressible flow, where B is a constant. Determine the pressure associated with this velocity field.

2023

1) Determine under what conditions the velocity field $u = c(x^2 - y^2), v = -2cxy,\ w = 0$ is a solution to the Navier-Stokes momentum equations. Assuming that the conditions are met, determine the resulting pressure distribution, when z ' is up and the external body forces are $B_x = 0 = B_y, B_z = -g$.

2) In a fluid motion, there is a source of strength $2m$ placed at $z = 2$ and two sinks of strength m are placed at $z = 2 + i$ and $z = 2 - i$. Find the streamlines.

2022

1) The velocity components of an incompressible fluid in spherical polar coordinates (r, θ, ψ) are $(2Mr^{-3}\cos\ \theta, Mr^{-3}\sin\ \theta, 0)$, where M is a constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of the streamlines.

   NOTE : UPSC Original paper has $Mr^{-2}sin\ \theta$, it should be $Mr^{-3}sin\ \theta$

2) Two-point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-\frac{k}{2}$ is situated at the origin. Show that the fluid motion is stationary and also find the equations of streamlines. If the streamlines, which pass through the stagnation points, meet the x-axis at $(\pm b, 0)$, then show that $3\sqrt{3}(b^2 - a^2)^2 = 16a^3 b$

3) Verify that $w = ik\log\{(z - ia)/(z + ia)\}$ is the complex potential of a steady flow of fluid about a circular cylinder, where the plane $y = 0$ is a rigid boundary. Find also the force exerted by the fluid on unit length of the cylinder.

2021

1) Show that $\vec{q} = \frac{\lambda(-y\hat{\imath}+x\hat{\jmath})}{x^2+y^2}, (\lambda = \text{constant})$ is a possible incompressible fluid motion. Determine the streamlines. Is the kind of the motion potential? If yes, then find the velocity potential.

2) Show that for the complex potential $\tan^{-1} z$, the streamlines and equipotential curves are circles. Find the velocity at any point and check the singularities at $z = \pm i$.

3) What arrangements of sources and sinks can have the velocity potential $= \log_e\left(z - \frac{a^2}{z}\right)$? Draw the corresponding sketch of the streamlines and prove that two of them subdivide into the circle $r = a$ and the axis of y.

2020

1) A velocity potential in a two-dimensional fluid flow is given by $\phi(x, y) = xy + x^2 - y^2$ find the stream function for this flow.

2) Two sources of strength $\frac{m}{2}$ are placed at the points $(\pm a, 0)$. Show that at any point. Show that at any point on the circle $x^2 + y^2 = a^2$ the velocity is parallel to the y axis and is inversely proportional to y.

2019

1) A sphere of radius R whose centre is at rest, vibrates radially in an infinite incompressible fluid of density ρ, which is at rest at infinity, if the pressure at infinity is Π, so that the pressure at the surface of the sphere at time t is $\Pi + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}$.

2) Two sources, each of strength are places at the points $(-a, 0), (a, 0)$ and a sink of strength at origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2a^2)/(r_1 r_2 r_3)$ where $r_1 r_2$ and r_1 re the distances of the points from the source and sings respectively.

2018

1) For an incompressible fluid flow, two components of velocity (u, v, w) are given by u $= x^2 + 2y^2 + 3z^2$ $v = x^2y - y^2z + zx$. Determine the third component w so that they satisfy the equation of continuity. Also, find the z-component of acceleration.

2) For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2y - xy^2 + \frac{1}{3}(x^3 - y^3)$.Determine the velocity components along the directions x and y. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not.

3) A thin annulus occupies the region. $0 \leq a \leq r \leq b, 0 \leq \theta \leq 2\pi$ The faces are insulated, Along the inner edge the temperature is maintained $0°$, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$ where K is a constant. Determine the temperature distribution in the annulus.

2017

1) A stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion is assumed to steady and diverging from the vertex of the cone, then prove that $\frac{v}{V} = \frac{D^2}{d^2}e^{(u^2-v^2)/2K}$, where K is the pressure divided by the density and is constant.

2) If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right), r^2 = x^2 + y^2 + z^2$, then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$. Further, determine the streamlines.

2016

1) Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$ possess vorticity, where $\vec{q}(u, v, w)$ is the velocity in the Cartesian frame $\vec{r}(x, y, z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9, z = 0$?

2) A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{i}$ Show that he velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\,\theta$, where $OP = r$ and θ is the angle which $\overrightarrow{OP}$ makes with the direction $\vec{i}$. Find the deferential equation of the streamlines and show that the lie on the surfaces $Ur^2\sin^2\,\theta - 2m\cos\,\theta =$ constant.

3) The space between two concentric spherical shells of radii $a, b(a < b)$ is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y - direction, then show has the initial motion of the liquid is given by velocity potential $\phi = \frac{\left\{a^3U\left(1+\frac{1}{2}b^3r^{-3}\right)x - b^3V\left(1+\frac{1}{2}a^3r^{-3}\right)y\right\}}{(b^3-a^3)}$, where $r^2 = x^2 + y^2 + z^2$, the coordinate being rectangular. Evaluate the velocity at any point of the liquid.

2015

1) Consider a uniform flow U_0 in the positive x-direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points.

2) In an axis symmetric motion, show that stream function exists due to equation of continuity. Express the velocity components of the stream function. Find the equation satisfied by the stream function if the flow is irrotational.

2014

1) Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$, determine the streamlines.

2) Find Navier-Stokes equation for steady laminar flow of a viscous incompressible fluid between two infinite parallel plates.

2013

1) Prove that the necessary and sufficient conditions that the vortex lines may be at right angles to the stream lines are $u, v, w = \mu\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$ where μ and ϕ are function of x, y, z, t.

2) If fluid fills the region of space on the positive side of the x-axis, which is a right boundary and if there be a sources m at the point $(0, a)$ and an equal sink at $(0, b)$ and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\frac{\pi\rho m^2(a-b)^2}{\{2ab(a+b)\}}$ where ρ is the density of the fluid.

3) If n rectilinear vortices of the same strength K are symmetrically arranged as generators of a circle cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^2}{(n-1)\mathrm{K}}$. Find the velocity at any point of the liquid.

2012

1) A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

2) Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q} \to 0$ as $r \to \infty$, find the surfaces of constant speed.

2011

1) An infinite row of the equidistance rectilinear vortices are at distance a apart. The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function.

2010

1) In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solution of Laplace's equation.

2) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distances from its axis, show that the path of each vortex is given by the equation $(r^2\sin^2\theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2\sin^2\theta$, θ being measured from the line through the centre perpendicular to the joint of the vortices.

2009

1) Two sources, each of strength m are placed at the point $(-a, 0), (a, 0)$ and a sink of strength $2m$ is at the origin. Show that the stream lines are the curves: $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2ma^2)(r_1 r_2 r_3)$ where $r_1 r_2$ and r_3 are the distance of the points from the source and the sink.

2) An infinite mass of fluid is acted on by a force $\frac{\mu}{r^{3/2}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = C$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{\frac{1}{2}} \cdot C^{\frac{5}{4}}$.

2008

1) If the velocity potential of a fluid is $\phi = \frac{1}{r^3} z\tan^{-1}\left(\frac{y}{z}\right), r^2 = x^2 + y^2 + z^2$ then show that the stream lines lie on the surfaces $x^2 + y^2 + z^2 = c(x^2 + y^2)^{2/3}, c$ being a constant.

2) Let the fluid fills the region $x \geq 0$ (right half of $2d$ plane). Let a source α be $(0, y_1)$ and equal sink at $(0, y_2), y_1 > y_2$. Let the pressure be same as pressure infinity i.e., p_0. Show that the resultant pressure on the boundary (y-axis) is $\pi\rho\alpha^2(y_1 - y_2)^2/2y_1y_2(y_1 + y_2), \rho$ being the density of the fluid.

2007

1) Show that $\left(\frac{x^2}{a^2}\right)\cos^2 t + \left(\frac{y^2}{b^2}\right)\sec^2 t = 1$ is a possible form for the boundary surface of a liquid.

2) A thin plate of very large area is placed in a gap of height h with oils of viscosities μ' and μ'' on the two sides of the plate. The plate is pulled at a constant velocity V. Calculate the position of the plate so that (i)The shear force on the sides of the two sides of the plate is equal (ii) The force required to drag the plate is minimum. [End effects are neglected]

2006

1) A steady Inviscid incompressible flow has a velocity field $u = fx, v = -fy, w = 0$, where f is a constant. Derive an expression for the pressure field $p(x, y, z)$, if the pressure $p(0,0,0) = p_0$ and $F = -giz$.

2) Liquid is contained between two parallel planes; the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated; prove that if p be the pressure at the outer surface, the initial pressure at any point on the liquid distant r from the centre is $p\frac{\log r - \log b}{\log a - \log b}$.

2005

1) State the conditions under which Euler's equation of motion can be integrated show that $-\frac{\partial\phi}{\partial t} + \frac{1}{2}q^2 + V\int\frac{dp}{\rho} = F(t)$ where the symbols have their usual meaning.

2) Prove that the necessary and sufficient condition for vortex lines and stream to be at right angles to each other is that $u = \mu\frac{\partial\phi}{\partial x}, v = \mu\frac{\partial\phi}{\partial y}, w = \mu\frac{\partial\phi}{\partial z}$ where μ and ϕ are functions of x, y, z and t.

2004

1) In an incompressible fluid the vorticity at every point is constant in magnitude and direction. constant, in magnitude and direction. Do the velocity components satisfy the Laplace equation? Justify.

2) The space between two infinitely long coaxial cylinder of radii a and $b(b > a)$ respectively is filled by a homogeneous fluid of density ρ. The inner cylinder is suddenly moved with velocity v perpendicular to this axis, the outer being kept fixed. Show that the resulting impulsive pressure on a length l of inner cylinder is $\pi\rho a^2 l \frac{b^2+a^2}{b^2-a^2} v$

2003

1) For an incompressible homogeneous fluid at the point (x, y, z) the velocity distribution is given by $u = -\frac{c^2 y}{r^2}, v = \frac{c^2 x}{r^2}, w = 0$; where r denotes the distance from z-axis. Show that it is a possible motion and determine the surface which is orthogonal to stream line.

2) Two sources, each of strength m are placed at the points $(-a, 0)$ and $(a, 0)$ and a sink of strength $2m$ is placed at the origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$, where λ is a variable parameter. Also show that fluid speed at any point is $\frac{2ma^2}{r_1 r_2 r_3}$ where r_1, r_2 and r_3 are respectively the distance of the point from the sources and sink.

3) An infinite mass of fluid is acted upon by a force $\mu r^{\frac{-3}{2}}$ per unit mass directed the origin. If initially the fluid is at a rest and there is a cavity in the form of a sphere $r = c$ in it. Show that the cavity will be filled up after an interval of time $\left\{\frac{2}{5\mu}\right\}^{\frac{1}{2}} c^{\frac{5}{4}}$.

2002

1) Show that the velocity potential $\phi = \frac{1}{2}a(x^2 + y^2 - 2z^2)$ satisfies the Laplace equation, and determine the stream lines.

2) Show that: $u = \frac{-2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational?

3) Prove that $\left(v\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2\psi = \frac{\partial(\psi, \nabla^2\psi)}{\partial(x,y)}$ where V is the kinematic viscosity of the fluid and ψ is the stream function for a two-dimensional motion of a viscous fluid.

2001

1) If the velocity distribution of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{kz^2-r^2}{r^5},\right)$ then determine the parameter k such that it is a possible motion. Hence find its velocity potential.

2) Show that the velocity distribution in axial flow of viscous incompressible fluid along a pipe of annular crosssection radii $r_1 < r_2$, is given by $\omega(r) = \frac{1}{4\mu}\frac{dp}{dz}\left\{r^2 - r_1^2 + \frac{r_2^2-r_1^2}{\log\left(\frac{r_2}{r_1}\right)}\log\left(\frac{r}{r_2}\right)\right\}$.