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TEST SERIES FOR UPSC MATHEMATICS MAINS EXAM 2024

Free Open Test 1 - PAPER 1

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Date :	Start Time: Closing Time:

Section A			Section B			Remarks
Q.No	Max Marks	Marks Obtained	Q.No	Max Marks	Marks Obtained	Please ADD Extra Paper, when it is required.
1a			5a			
1b			5b			
1c			5c			
1d			5d			
1e			5e			
2a			6a			
2b			6b			
2c			6c			
3a			7a			
3b			7b			
3c			7c			
4a			8a			
4b			8b			
4c			8c			
Total						

Section A

1a) Using elementary transformations, find the inverse of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$ (10)

1b) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$ (10)

1c) Test for convergence

$$\int_0^\infty \frac{\sin\ x^m}{x^n} dx$$

(10)

1d) Let $P(x)$ be a polynomial with real coefficients. Let $a, b \in R, a < b$, be two consecutive roots of $P(x)$. Show that there exists ' c ' such that $a \le c \le b$ and $P'(c) + 100P(c) = 0.$ (10)

1e) A variable plane makes intercepts on the co-ordinate axes the sum of whose squares is constant and equal to k^2. Show. that the locus of the foot of the perpendicular from the origin to the plane is $(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2) = k^2.$ (10)

2a) Find a 3×3 real matrix A such that $Au_1 = u_1, Au_2 = 2u_2, Au_3 = 3u_3$, where

$$u_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, u_2 = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, u_3 = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$$

(20)

2b) Find the length of the shortest distance between the z-axis and the line

$$x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4.$$ (15)

2c) Trace $y(1 - x^2) = x^2$. (15)

3a) Let $T: R^4 \to R^3$ be the linear transformation given by the formula

$$T(x_1, x_2, x_3, x_4) = (4x_1 + x_2 - 2x_3 - 3x_4\ , 2x_1 + x_2 + x_3 - 4x_4\ , 6x_1 - 9x_3 + 9x_4)$$

(a) Find a basis for ker (T).
(b) Find a basis for $R(T)$.
(c) Verify the dimension theorem.

(20)

3b) Find the maximum and minimum value of

$$f(x) = \frac{40}{3x^4 + 8x^3 - 18x^2 + 60}.$$

(15)

3c) Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $u^2/a + v^2/b + w^2/c = d.$ (15)

4a) Find that the area included between the parabolas $y^2 = 4a(x + a)$ and $y^2 = 4b(b - x)$. (15)

4b) Find the rank, signature and index of the quadratic form $2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 4x_1x_3 - 8x_2x_3$ by reducing it to canonical form or normal form. Also write the linear transformation which brings about the normal reduction. (15)

4c) Prove that the equation $x^2 + y^2 + z^2 + yz + zx + xy + 3x + y + 4z + 4 = 0$ represents an ellipsoid the squares of whose semi-axes are 2,2,$\frac{1}{2}$. Show that its principal axis is given by $x + 1 = y - 1 = z + 2$. (20)

Section B

5a) Solve : $(D^2 + 10D + 29)y = xe^{5x} + \sin\ 2x.$ (10)

5b) Use Laplace to solve $(D^2 + 1)y = \sin t \cos\ 2t, t > 0$, if $y = 1, \mathrm{D}y = 0$ when $t = 0$ (10)

5c) Show that the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley is $a\left\{\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right\}$. (10)

5d) A particle describes the cardioid $r = a(1 + \cos\ \theta)$ under a central force to the pole. Find the law of force. (10)

5e) Find the directional derivative of $x^2y^2z^2$ at the point $(1,1,-1)$ in the direction of the tangent to the curve $x = e^t, y = \sin\ 2t + 1, z = 1 - \cos\ t$ at $t = 0$. (10)

6a) Solve $x^2 \frac{d^2y}{dx^2} - 2(x^2 + x)\frac{dy}{dx} + (x^2 + 2x + 2)y = 0.$ (20)

6b) A ladder 30 m long rests with one end against a smooth vertical wall and with the other end on the ground which is rough, the co-efficient of friction being $\frac{1}{2}$. Find how high a man whose weight is four-times that of the ladder can ascend before it begins to slip, the foot of the ladder being 6 m from the wall. (15)

6c) A ball is projected so as just to clear two walls the first of height a at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on the horizontal plane is $(a^2 + ab + b^2)/(a + b)$, and that the angle of projection exceeds $\tan^{-1}\ 3$. (15)

7a) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} + (1 - \cot\ x)\frac{dy}{dx} - y\cot\ x = \sin^2\ x$$ (20)

7b) Verify Divergence Theorem, given that $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$ (15)

7c) A tangent is drawn to a given curve at some point of contact. B is a point on the tangent at a distance 5 units from the point of contact. Show that the curvature of the locus of the point B is $\frac{\left[25k^2\tau^2(1+25k^2)+\left\{k+5\frac{dk}{ds}+25k^3\right\}\right]^{1/2}}{(1+25k^2)^{3/2}}$. (15)

8a) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$, given that $x + \frac{1}{x}$ is one integral. (15)

8b) Verify Stoke's Theorem for the function $\bar{F} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$, where C is the unit circle in xy-plane bounding the hemisphere $z = \sqrt{(1 - x^2 - y^2)}$. (20)

8c) Find general and singular solutions of $9p^2(2 - y)^2 = 4(3 - y)$. (15)