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TEST SERIES FOR UPSC MATHEMATICS MAINS EXAM 2024

Free Open Test 2 - PAPER 2

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Date :	Start Time: Closing Time:

Section A			Section B			Remarks
Q.No	Max Marks	Marks Obtained	Q.No	Max Marks	Marks Obtained	Please ADD Extra Paper, when it is required.
1a			5a			
1b			5b			
1c			5c			
1d			5d			
1e			5e			
2a			6a			
2b			6b			
2c			6c			
3a			7a			
3b			7b			
3c			7c			
4a			8a			
4b			8b			
4c			8c			
Total						

Section A

1a) Let a be an element in a group such that Ord $(a) = 20$. Find $\mathrm{Ord}\ (a^6)$ and $\mathrm{Ord}\ (a^{13})$ (10)

1b) Show that $\mathbf{Z}_5(x)$ is a UFD. Is $x^2 + 2x + 3$ will be reducible over $\mathbf{Z}_5(x)$?

(10)

1c) Find the relative extrema of the function

$$f(x, y) = 4y^3 + x^2 - 12y^2 - 36y + 2.$$

(10)

1d) If $u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$, find the corresponding analytic function $f(z)=u+iv$. (10)

1e) Solve the following LPP by simplex method

$$\max. Z = 3x_1 + 2x_2 + 5x_3$$

subject to the constraints

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

and $x_1, x_2, x_3 \geq 0$

(10)

2a) Let R be the ring of all the real-valued continuous functions on the closed unit interval. Show that $M = \left\{f \in R : f\left(\frac{1}{3}\right) = 0\right\}$ is a maximal ideal of R. (15)

2b) A scope probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth atmosphere and its surface begins to heat. After one hour the temperature at the point (x, y, z) on the sunface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe surface.

(20)

2c) A company has 4 machines on which to do 3 jobs. Each job can be assigned to one and only one member. The cost of each job on each

	Machine			
Job↓	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

machine is given in the following table

What are the job assignments that will minimize the cost?

(15)

3a) If $o(G) = pq, p$ and q are distinct primes and $p < q$. show that if p does not divide $(q - 1)$, then G is cyclic. (15)

3b) If a function f is continuous in $[0,1]$, show that

$$\lim_{n\to\infty} \int_0^1 \frac{nf(x)}{1+n^2x^2}\,dx = \frac{\pi}{2} f(0)$$

(15)

3c) Find the Taylor series of the following functions and their radii of convergence:

(a) $z\sinh(z^2)$ at $z = 0$;

(b) e^z at $z = 2$;

(c) $\frac{z^2+z}{(1-z)^2}$ at $z = -1$.

(20)

4a) Consider a function $f(x)$ whose second derivative $f''(x)$ exists and is continuous on $[0,1]$. Assume that $f(0) = f(1) = 0$ and suppose that there exists $A > 0$ such that $|f''(x)| \leq A$ for $x \in [0,1]$.

Show that $\left|f'\left(\frac{1}{2}\right)\right| \leq \frac{A}{4}$ and $|f'(x)| \leq \frac{A}{2}$ for $0 < x < 1$.

(20)

4b) Prove that $\int_0^\infty \sin(x^2)dx = \int_0^\infty \cos(x^2)dx = \frac{\sqrt{2\pi}}{4}$. (15)

4c) A company has factories at F_1, F_2, and F_3 that supply products to warehouses at W_1, W_2 and W_3. The weekly capacities of the factories are 200,160 and 90 units, respectively. The weekly warehouse requirements are 180, 120 and 150 units, respectively. The unit shipping costs (in

		Warehouse			
		W_1	W_2	W_3	*Supply*
Factory	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
	Demand	180	120	150	450

rupees) are as follows:

Determine the optimal distribution for this company in order to minimize its total shipping cost. (15)

Section B

5a) Form the partial differential equation by eliminating the arbitrary function ϕ from $\phi\left(\frac{y}{x}, x^2 + y^2 + z^2\right) = 0$. (10)

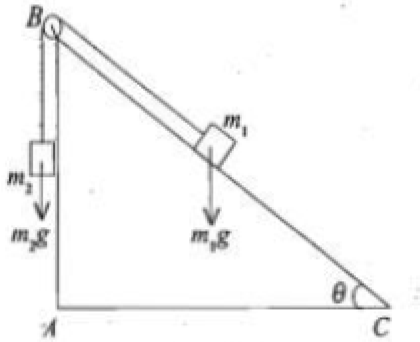
5b) Determine the decimal number represented by the following binary numbers $(110101.0101)_2, (26.6)_{16}, (36.34)_8$, (BAFC.C) $)_{16}$ (10)

5c) Obtain Newton-Raphson formula to find:

(i) square root, (ii) r^{th} root of a given number (iii) inverse, (iv) inverse square root of a given number. (10)

5d) The stream function for a two-dimensional incompressible flow is $\psi = \frac{ax^2}{2} + bxy - \frac{cy^2}{2}$, where a, b and c are known constants. Find the condition for the flow to be irrotational and thus find the velocity potential for the flow. (10)

5e) An inextensible string of negligible mass hanging over a smooth peg B[Fig] connects one mass m_1 on a frictionless inclined plane of angle θ to another mass m_2. Using D'Alembert's principle, prove that the masses



will be in equilibrium, if $\sin\,\theta = \frac{m_2}{m_1}$ (10)

6a) Find the surface passing through the parabolas

$$z = 0, y^2 = 4ax \text{ and } z = 1, y^2 = -4ax$$

and satisfying the equation $xr + 2p = 0$. (15)

6b) Derive Simpson 3/8. Rule and also derive their Error Formula (20)

6c) A spring of mass M and spring constant k, is hung vertically. Another mass, m, is suspended from it. Write down the Lagrangian of the system and show that the system will execute S.H.M. of period $2\pi\sqrt{\frac{(M/3)+m}{k}}$

(15)

7a) A dynamical system has the Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + b q_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2,$$

where a, b, k_1, and k_2 are constants. Find the equations of motion in the Hamiltonian formalism. (20)

7b) Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution. (15)

7c) Express the following function in sum of minterms and product of maxterms. $F(A, B, C, D) = B'D + A'D + BD$ (15)

8a) A tightly stretched string with fixed ends at $x = 0$ and $x = l$ is initially in equilibrium position. It is set vibrating by giving to each of its point an initial velocity $b\sin^3(\pi x/l)$. Find the displacement of the string at any time t. (20)

8b) State the sufficient condition for convergence of the Gauss-Seidel iteration method and solve the following system of equations by using this method $5x + 4y - 10z = 65$ $4x + 8y + 3z = 155$ $6x + y + z = 105$ (15)

8c) There is a doublet at $(c, 0)$ in a 2-dimensional flow. A cylinder of radius a ($\mathrm{a} < c$) with z-axis as axis of the cylinder was introduced into the flow. Find the complex potential and image system for the flow. (15)