

# **Best Coaching for UPSC MATHEMATICS**

# UPSC MATHEMATICS STUDY MATERIAL BOOK- 15 Flow Charts

# **Important Links**

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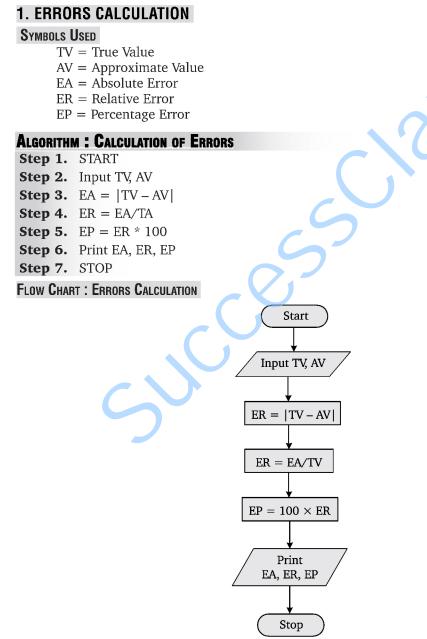
FLOW CHARTS ALGORITHMS\_opt

#### ERROR AND APPROXIMATIONS



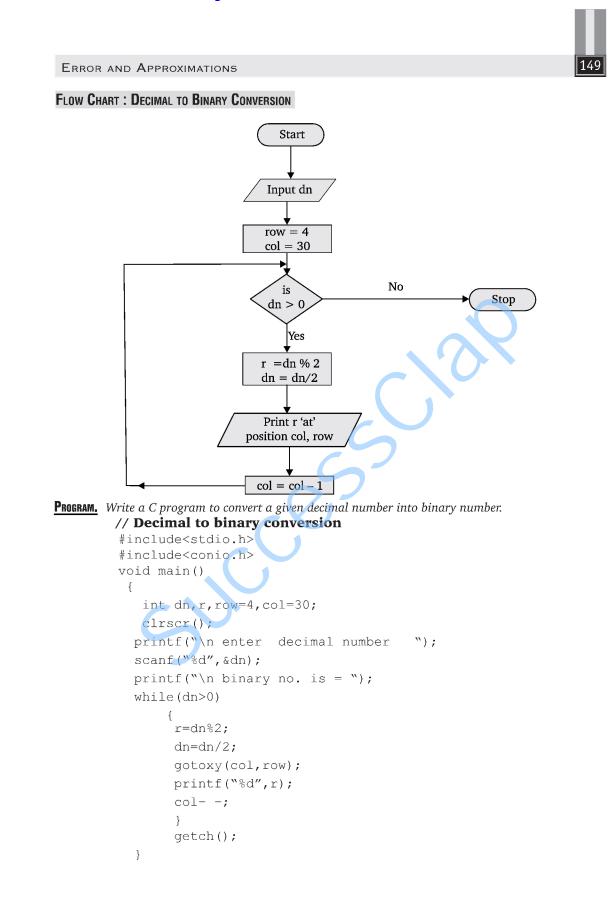
# COMPUTATIONAL TECHNIQUE LAB

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```
NUMERICAL METHODS IN SCIENCE AND ENGINEERING
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PROGRAM: Write a program to calculate the errors.
    //error calculation
         #include<stdio.h>
         #include<conio.h>
         #include<math.h>
         void main()
         {
            float tv, av, er, ep, ea;
            clrscr();
            printf("\n enter true value (n'');
            scanf( ``%f",&tv);
            printf("\n enter approximate value\n");
            scanf( ``%f",&av);
            ea=fabs(tv-av);
            er=ea/tv;
            ep=100*er;
            printf("\n\n absolute error= %e_",ea);
            printf("\n\n relative error= %e ",er);
            printf("\n\n percentage error= %e ",ep);
            getch();
            }
         Output: ERROR CALCULATION
         enter true value
         37.46235
         enter approximate value
         37.46
         absolute error= 2.349854e-03
         relative error= 6.272574e-05
         percentage error= 6.272574e-03
2. CONVERSION OF DECIMAL TO BINARY NUMBER
SYMBOLS USED
     bn = binary number
     dn = decimal number
     r = remainder
ALGORITHM : DECIMAL TO BINARY CONVERSION
Step 1. START
Step 2. Input dn
Step 3. row = 4, col = 30
Step 4. Perform steps 5 to 8 while (dn > 0)
Step 5. r = dn\%2
Step 6. dn = dn/2
Step 7. Print r 'at' col, row
Step 8. col = col - 1
```

Step 9. STOP





NUMERICAL METHODS IN SCIENCE AND ENGINEERING

**Output: DECIMAL TO BINARY CONVERSION** enter decimal number 99

binary no. is = 1100011

## 3. CONVERSION OF BINARY TO DECIMAL NUMBER

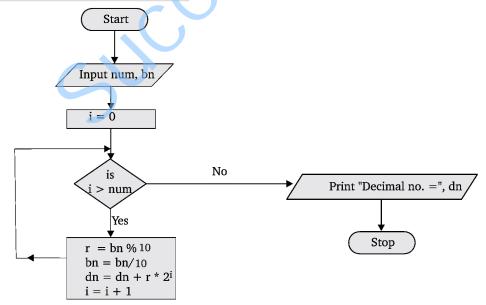
### Symbols Used

- bn = binary number
- dn = decimal number
- num = number of digits in a binary number
  - r = remainder

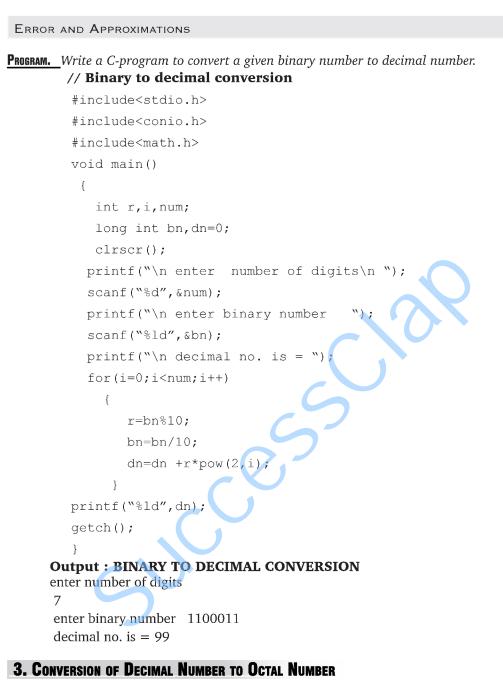
## **ALGORITHM : BINARY TO DECIMAL CONVERSION**

Step 1. Start Step 2. Input num, bn Step 3. i = 0Perform step 5 to 8 while (i < num) Step 4. Step 5. r = bn % 10 Step 6. bn = bn/10 $dn = dn + r * (2^1)$ Step 7. Step 8. i = i + 1Step 9. Print "decimal no = ", dn Step 10. Stop

FLOW CHART : BINARY TO DECIMAL NUMBER



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#### SYMBOLS USED

dn = decimal number on = octal number

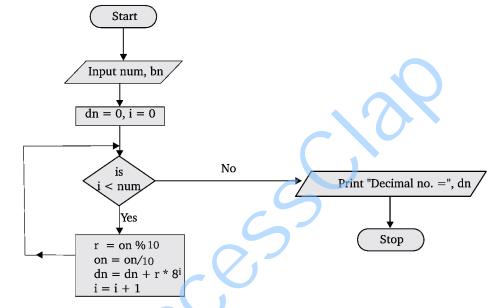
## **ALGORITHM : CONVERT DECIMAL NUMBER INTO EQUIVALENT OCTAL NUMBER**

1

Step 1.	Start
Step 2.	Input dn
Step 3.	on = 0, i =

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Step 4.	Perform steps 5 to 8 while $(dn > 0)$
Step 5.	r = dn % 8
Step 6.	dn = dn/8
Step 7.	$on = on + r^* i$
Step 8.	$i = i^* \ 10$
Step 9.	Print "Equivalent octal number" on,
Step 10.	Stop

FLOW CHART : CONVERSION OF DECIMAL NUMBER TO OCTAL NUMBER



**PROGRAM :** Write a C program to convert a decimal number into octal number. // **Decimal to octal conversion** 

```
#include<stdio.h>
#include<conio.h>
void main()
£
int dn,r,on=0,i=1;
clrscr();
printf("\n enter decimal number
                                  ");
scanf("%d",&dn);
printf("\n octal no. is = ");
while(dn>0)
{
         r=dn%8;
         dn=dn/8;
         on=on+r*i;
         i=i*10;
}
printf(``%d",on);
```

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ERROR AND APPROXIMATIONS

getch();

}

#### **Output : DECIMAL TO OCTAL CONVERSION**

enter decimal number 9876 Octal no. is = 23072

### 4. CONVERSION OF GIVEN OCTAL NUMBER INTO AN EQUIVALENT DECIMAL NUMBER

#### SYMBOLS USED

dn = decimal number on = octal number

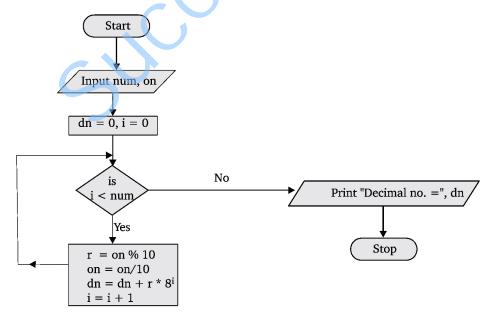
num = number of digits in Octal number

#### Algorithm : Convert Octal number into Decimal number

- Step 1. START
- Step 2. Input "Enter number of digits", num
- **Step 3.** Input "Enter Octal number", on
- **Step 4.** i = 0; dn = 0
- **Step 5.** Perform steps 6 to 8 while (i < num)
- **Step 6.** r = on % 10
- **Step 7.** on = on/10
- **Step 8.**  $dn = dn + r * 8^{i}$
- **Step 9.** Print "decimal number = ", dn

#### Step 10. STOP

FLOW CHART : CONVERSION OF OCTAL NUMBER INTO DECIMAL NUMBER



```
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                                       NUMERICAL METHODS IN SCIENCE AND ENGINEERING
PROGRAM: Write a C program to convert a given octal number into a decimal number.
     // Octal to decimal conversion
         #include<stdio.h>
         #include<conio.h>
         #include<math.h>
         void main()
          {
         int r,i,num;
         int on, dn=0;
         clrscr();
         printf("\n enter number of digits\n ");
         scanf("%d",&num);
         printf("\n enter octal number
                                             ");
         scanf("%d",&on);
         printf("\n decimal no. is = ");
         for(i=0;i<num;i++)</pre>
         {
                 r=on%10;
                 on=on/10;
                 dn=dn+r*pow(8,i);
         }
         printf(``%d",dn);
         getch();
         }
      Output : OCTAL TO DECIMAL CONVERSION
      enter number of digits 4
      enter octal number 1727
      decimal no. is = 983
                                  жжжжжж
```



# **1. BISECTION METHOD**

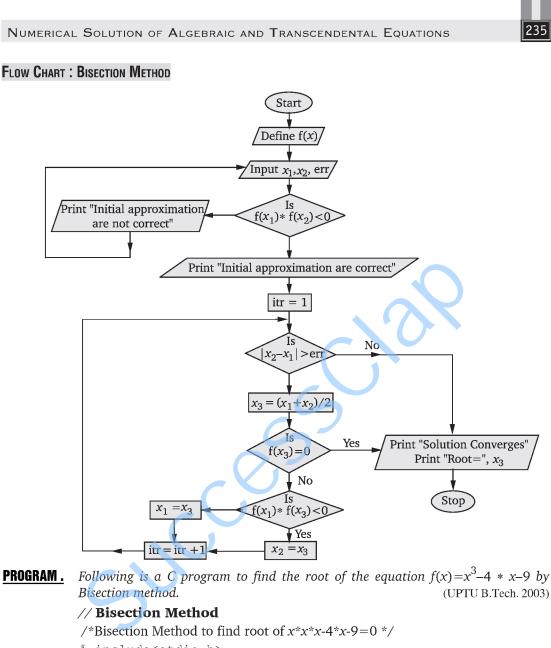
#### SYMBOL USED

 $x_1, x_2$  = Initial approximations in which root lies.

- err = allow error
- $x_3$  = New approximation of the root in each iteration
- itr = a counter which keeps track of the no. of iterations performed.

## **ALGORITHM : BISECTION METHOD**

Step 1.	Start
Step 2.	Define <i>f</i> ( <i>x</i> )
Step 3.	Input $x_1, x_2$ , err
Step 4.	If $(f(x_1) * (f(x_2)) < 0$
	print "Initial approximations are correct"
	else
	print "Initial approximations are not correct"
	go to step 3
Step 5.	
Step 6.	Perform steps 7 to 10 while $( x_2 - x_1  > err)$
Step 7.	$x_3 = (x_1 + x_2)/2$
Step 8.	$\text{if } f(x_3) = 0$
	Print "Solution converges in iteration", i
	Print "Root =", $x_3$
	go to step 13
Step 9.	If $(f(x_1) * (f(x_3)) < 0$
	$x_2 = x_3$
	else
	$x_1 = x_3$
Step 10.	itr = itr + 1
Step 11.	Print "Solution converges in iteration", i
Step 12.	Print "Root=", x <sub>3</sub>
Step 13.	Č.



- # include<stdio.h>
- # include<conio.h>
- # include<math.h>
- # include<process.h>

```
float f(float x)
{
    return(x*x*x-4*x-9);
}
```

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```
NUMERICAL METHODS IN SCIENCE AND ENGINEERING
 void main()
ł
  clrscr();
int itr=1;
float x1,x2,x3, err;
start: printf("Enter the value of x1, x2,"
                      "allowed error\n");
       scanf("%f%f%f", &x1,&x2,&err);
if(f(x1) * f(x2) < 0)
    printf("\n initial approximations are correct\n");
else
   { printf("\n initial approximations are not correct\n");
      goto start;
    }
while(fabs(x2-x1)>err)
{
   x3=(x1+x2)/2;
   if(f(x3) == 0)
      {
      printf("\nsolution converges in iteration=%d",itr);
      printf("\n Root=%f",x3);
      exit(0);
      }
     if(f(x1)*f(x3)<0)
        x^2=x^3;
     else
        x1=x3;
   itr++;
    printf("\n solution converges in iteration=%d",itr);
    printf("\n Root=%f",x3);
    getch();
}
Output: BISECTION METHOD TO FIND ROOT OF x*x*x-4*x-9=0
```

```
Enter the value of x1, x2, allowed error
3 2 .0005
initial approximations are correct
solution converges in iteration =12
Root=2.7065
```

237 NUMERICAL SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS LAB ASSIGNMENT : BISECTION METHOD **1.** Write a C program to find the root of the equation by using Bisection method correct to two places of decimal.  $f(x) = x^3 - x - 11 = 0$ **Hint:** Define a function f(x) = x \* x \* x - x - 11 $x_1 = 2, x_2 = 3, \text{ err} = 0.5 \times 10^{-2} = 0.005$ Input: **Output:** Root = 2.38 **2.** Write C program to find root for the given function by Bisection method correct to two decimal places.  $f(x) = 3x - \sqrt{1 + \sin x}$  $f(x) = 3 * x - \text{sqrt} (1 + \sin (x))$ **Hint:** Define  $x_1 = 0, x_2 = 1 \text{ err} = 0.005$ Input : **Output:** Root = 0.39 3. Write a C program to find root using Bisection method correct to two decimal places for following function.  $f(x) = x^2 - 4x - 10 = 0$ f(x) = x \* x - 4 \* x - 10Hint: Define  $x_1 = 5, x_2 = 6$  and err = 0.005 Input Output Root = 5.7384. Write a C program to find root using Bisection method correct to two decimal places for following function.  $x \log_{10} x = 1.2$ **Hint:** Define  $f(x) = x * \log_{10} (x) - 1.2$ Input  $x_1 = 2, x_2 = 3$ , err = 0.005

Output Root = 2.74.

## 2. ITERATION METHOD

#### SYMBOLS USED

 $x_1$  = Initial approximations of the root

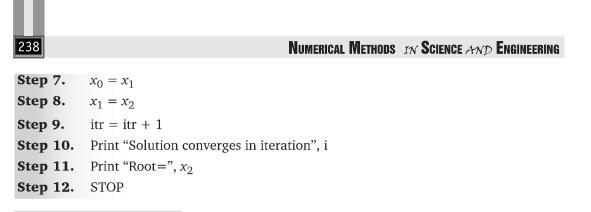
- $x_2$  = New approximation of root in each iteration
- $x_0$  = Used to store the previous value of  $x_1$

## **ALGORITHM : ITERATION METHOD**

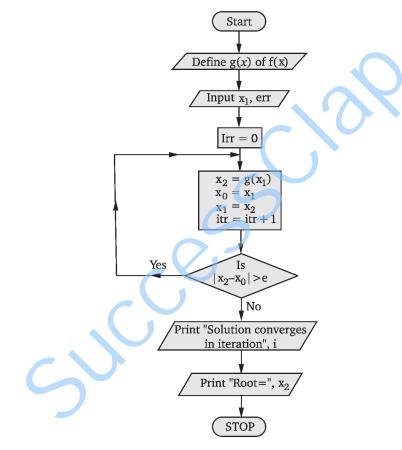
Step 1. Start

- Step 2. For given function f(x) defined g(x)
- Step 3. Input  $x_1$ , err
- Step 4. itr = 0
- Step 5. Perform steps 6 to 9 while  $(|x_2 - x_0| > err)$
- Step 6.  $x_2 = g(x_1)$





FLOW CHART : ITERATION METHOD



**PROGRAM.** Following is a C program to find the root of the equation  $f(x) = 2x - \log_{10} x - 7 = 0$  by using iteration method correct to 4 decimal places.

We have  $f(x) = 2x - \log_{10} x - 7 = 0$ It can be written as

$$x = \frac{1}{2}(\log_{10} x + 7)$$

Define

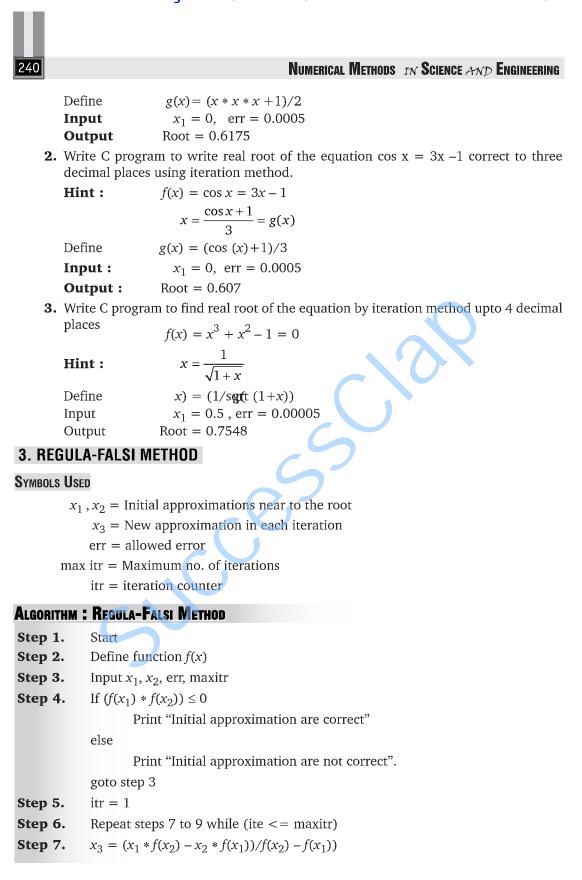
 $g(x) = (\log_{10}(x) + 7)/2$ 

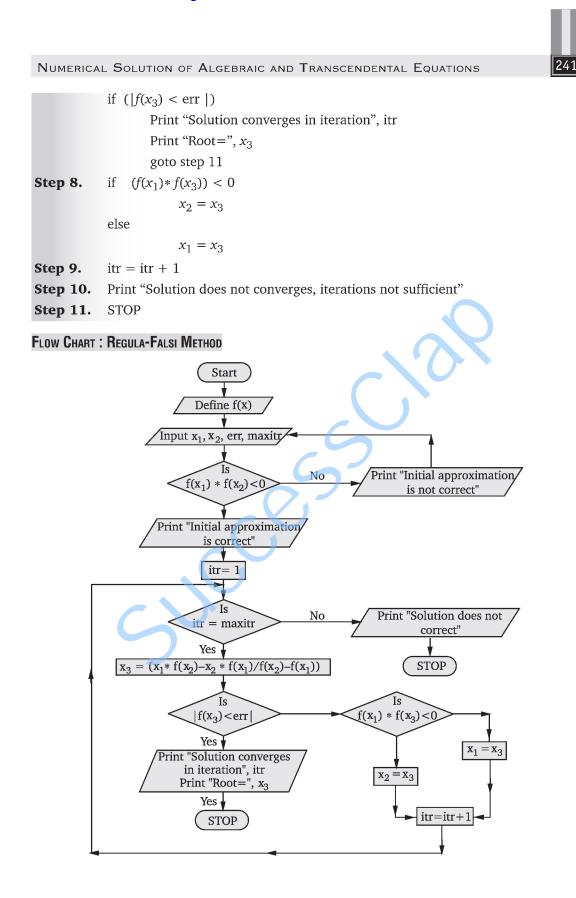
```
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NUMERICAL SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS
         Program for iteration method
         /*Iteration Method to find root of 2x-\log_{10}x-7=0 */
           # include<stdio.h>
           # include<conio.h>
           #include<math.h>
           #include<process.h>
         float g(float x)
          ł
               return ((\log 10(x) + 7)/2);
           }
          void main()
          ł
            clrscr();
          int itr=0;
         float x1,x2,xo, err;
                printf("Enter the value of x1, allowed errorn'');
                scanf("%f%f", &x1,&err);
            do
              {
                x2=q(x1);
                xo=x1;
               x1=x2;
               itr++;
                } while(fabs((x2-xo)>err));
                printf("\n solution converges in iteration =%d",itr);
                printf("\n Root=%f",x2);
               getch();
         Output: ITERATION METHOD TO FIND ROOT OF 2x-log10x-7=0
          Enter the value of x1, allowed error
              .00005
          3
          solution converges in iteration = 5
         Root= 3.789278
```

#### LAB ASSIGNMENTS : ITERATION METHOD

**1.** Write a C program to find root of the equation  $f(x) = x^3 - 2x + 1 = 0$  using iteration method correct to 3 decimal places.

Hint:  $f(x) = x^3 - 2x + 1 = 0$  $g(x) = x = \frac{x^3 + 1}{2}$ 





```
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                                       NUMERICAL METHODS IN SCIENCE AND ENGINEERING
PROGRAM.
          Following is a C program to find the root of the equation f(x) = \cos x - xe^x by using
          Regula-Falsi method.
                                                               (UPTU B.Tech. 2002)
           REGULA FALSI METHOD
          /* REGULA FALSI METHOD FOR \cos(x)-xe^{x} */
           #include<stdio.h>
           #include<conio.h>
           #include<math.h>
           float f(float x)
           ł
           return cos(x) - x + exp(x);
           }
           void main()
           {
                 int itr, maxitr;
                 float x1,x2,x3,err;
                 clrscr();
              start: printf("enter the value of x1, x2, allowed error,
           maximum iteration\n");
                 scanf("%f%f%f%d", &x1,&x2,&err, &maxitr);
                 if(f(x1) * f(x2) < 0)
                      printf("\n inital approximation are correct");
                  else
                   {
                    printf("\n inital approximation are not correct\n");
                      goto start;
                     }
              for(itr=1;itr<=maxitr;itr++)</pre>
                ł
                  x3 = (x1 + f(x2) - x2 + f(x1)) / (f(x2) - f(x1));
                 if(fabs(f(x3))<=err)</pre>
                 {
                   printf("\n Solutions converges in iterations =%d",
                     itr);
                    printf("\n Root=%f",x3);
                    getch();
                    exit(0);
                if (f(x1)*f(x3)<0)
                    x2=x3;
                else
                    x1=x3;
              }
              printf("\n Solution does not converge,"
                 "iterations not Sufficient\n");
               qetch();
               }
```

NUMERICAL SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS Output: REGULA FALSI METHOD FOR COS(X)-X\*eX enter the value of x1,x2, allowed error, maximum iteration 0 1 .00005 20 Solutions converges in iterations = 10 Root = 0.517748

#### Lab Assignments

**1.** Write C program to find root of the equation  $x^3 - 9x + 1 = 0$  by Regula-Falsi method.

Hint : Define f(x) = x \* x \* x - 9 \* x + 1Input  $x_1 = 2, x_2 = 3$ , err = 0.00005, maxitr = 8 Output Root = 2.9428

**2.** Write C program to find root of the equation  $xe^{x}-3 = 0$  by Regula-Falsi method correct to three decimal places.

**Input :**  $x_1 = 1, x_2 = 1.5, \text{ err} = 0.0005, \text{ maxitr} = 10$ **Output :** Root = 1.049

3. Write C program to find real root of the equation by Regula-Falsi method  $f(x) = x^2 - \log_e x - 12$ 

Hint : Define  $f(x) = x * x - \log (x) - 12$ Input :  $x_1 = 3, x_2 = 4, \text{ err } = 0.0005, \text{ maxitr } = 8$ 

## 4. NEWTON-RAPHSON METHOD

Symbols Used

 $x_1$  = Initial approximation of the root

 $x_2$  = New approximation of the root in each iteration

itr = iteration counter

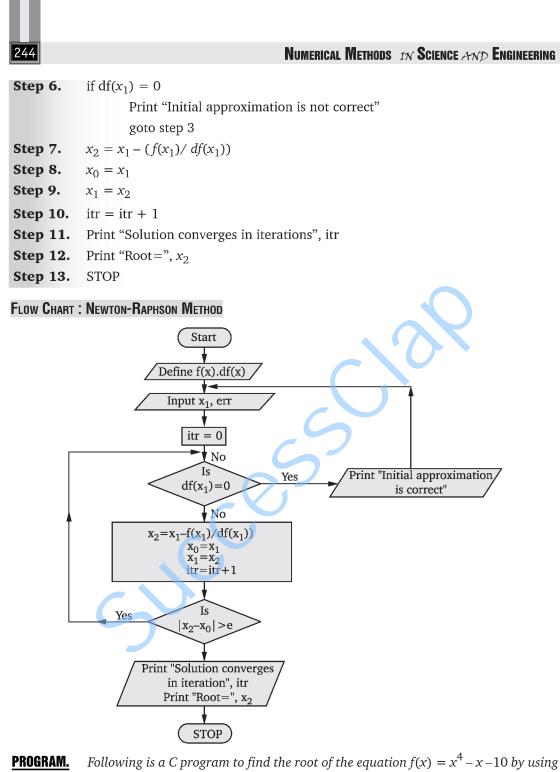
- err = allowed error
- $x_0 =$ old value of  $x_1$  and  $x_1$  is changed in each iteration

df(x) = first derivative of f(x)

#### Algorithm : Newton-Raphson Method

Step 1. Start

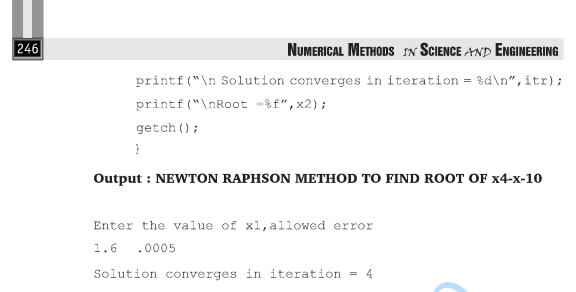
- **Step 2.** Define function f(x) and df(x)
- **Step 3.** Input  $x_1$ , err
- **Step 4.** itr = 0
- **Step 5.** Repeat steps 6 to 10 until ( $|x_2 x_0| < e$ )



**PROGRAM.** Following is a C program to find the root of the equation  $f(x) = x^4 - x - 10$  by using Newton-Raphson method.

We have  $f(x) = x^4 - x - 10$  $\Rightarrow \qquad df(x) = 4x^3 - 1$ 

```
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NUMERICAL SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS
         NEWTON RAPHSON METHOD
         /*NEWTON RAPHSON METHOD TO FIND ROOT OF x^4-x-10 */
         #include<stdio.h>
         #include<conio.h>
         #include<math.h>
         float f(float x)
          {
               return x*x*x*x-x-10;
          }
         float df (float x)
         {
               return 4*x*x*x-1;
          }
         void main ()
          ſ
               int itr=0;
               float x1,x2,err,x0;
               clrscr();
                       printf (\ n Enter the value of x1, allowed error
          start:
         n n'';
               scanf(``%f%f",&x1,&err);
               do
                  f(df(x1) == 0)
                    printf("\n Initial approximation is not correct");
                       goto start;
                      }
                x2=x1-f(x1)/df(x1);
                xo=x1;
                x1=x2;
                itr++ ;
               } while(fabs(x2-xo)>err);
```



Root=1.855585

#### LAB ASSIGNMENTS : NEWTON-RAPHSON METHOD

**1.** Write C program to find root of the equation  $f(x) = x^3 - 2x - 5$  correct to 3 decimal places by Newton-Raphson method.

Hint : Define f(x) = x \* x \* x - 2 \* x - 5and f(x) = 3 \* x \* x - 2Input  $x_1 = 2$ , err = 0.0005 Output Root = 2.09455

**2.** Write C program to find root of the equation  $x \log_{10} x = 1.2$  by Newton-Raphson method correct to four decimal places.

Hint : Define  $f(x) = x * \log_{10} (x) - 1.2$   $df(x) = \log_{10} (x) + 0.4343$ Input :  $x_1 = 2$ , err = 0.00005 Output : Root = 2.7408

**3.** Write C program to find square root of 12 correct to three decimal places by Newton-Raphson method.

Hint : Define  $f(x) = x^2 - 12 = 0$  f(x) = x \* x - 12 df(x) = 2 \* xInput :  $x_1 = 3.4$ , err = 0.0005 Output : Root = 3.464

NUMERICAL SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS



# 5. MULLER'S METHOD

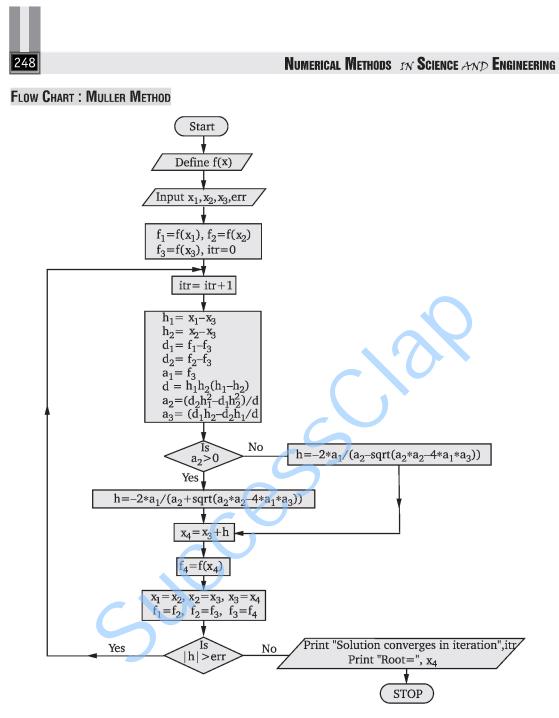
#### SYMBOLS USED

 $x_1, x_2, x_3 =$  Initial approximation

- $x_4$  = New iteration in each step
- err = allowed error
- itr = iteration counter

## **ALGORITHM : MULLER'S METHOD**

Step 1.	Start
Step 2.	Define $f(x)$
Step 3.	Input $x_1, x_2, x_3$ , err
Step 4.	$f_1 = f(x_1), f_2 = f(x_2), f_3 = f(x_3), \text{ itr } = 0$
Step 5.	Repeat steps 6 to 17 until ( $ h  < e$ )
Step 6.	itr = itr + 1
Step 7.	$h_1 = x_1 - x_3$ , $h_2 = x_2 - x_3$
Step 8.	$d_1 = f_1 - f_3, d_2 = f_2 - f_3$
Step 9.	$a_1 = f_1$
<b>Step 10.</b>	$d = h_1 * h_2 * (h_1 - h_2)$
Step 11.	$a_2 = (d_2 * h_1^2 - d_1 * h_2^2)/d$
Step 12.	$a_3 = (d_1 * h_2 - d_2 * h_1)/d$
Step 13.	if $(a_2 > 0)$
	$h = -2 * a_1 / (a_2 + \sqrt{a_2^2 - 4 * a_1 * a_3})$
	else
	$h = -2 * a_1 / (a_2 - \sqrt{a_2^2 - 4 * a_1 * a_3})$
Step 14.	$x_4 = x_3 + h$
Step 15.	$f_4 = f(x_4)$
Step 16.	$x_1 = x_2, x_2 = x_3, x_3 = x_4$
Step 17.	$f_1 = f_2, f_2 = f_3, f_3 = f_4$
Step 18.	Print "Solution converges in iteration", itr
Step 19.	Print "Root=", $x_4$
Step 20.	STOP



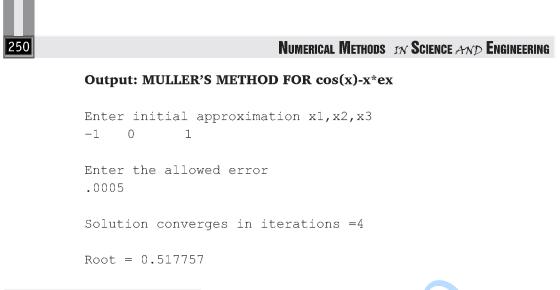
**PROGRAM.** Following is a C program to find the root of the equation  $f(x) = \cos x - xe^x$  by Muller's method.

#### **MULLER'S METHOD**

/\* MULLER'S METHOD cos(x)-x\*ex \*/

#include<stdio.h>
#include<conio.h>

```
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NUMERICAL SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS
          #include<math.h>
         float f(float x)
          {
          return \cos(x) - x^* \exp(x);
          }
         void main()
             {
               int itr=0;
               float x1, x2, x3, x4, err, d, d1, d2, h1, h2, a1, a2, a3, h;
               float f1, f2, f3, f4;
               clrscr();
             printf("\n Enter initial approximation x1, x2, x3 \n");
             scanf("%f%f%f",&x1,&x2,&x3);
             printf("\n Enter the allowed error n'');
             scanf("%f", &err);
             f1=f(x1), f2=f(x2), f3=f(x3);
              do
               {
                 itr++ ;
                 h1=x1-x3;
                 h2=x2-x3;
                 d1=f1-f3;
                 d2=f2-f3;
                 a1=f3;
                 d=h1*h2*(h1-h2);
                 a2=(d2*h1*h1-d1*h2*h2)/d;
                 a3 = (d1 * h2 - d2 * h1) / d;
                 if(a2>0)
                      h=-2*a1/(a2+sqrt(a2*a2-4*a1*a3));
                  else
                       h=-2*a1/(a2-sqrt(a2*a2-4*a1*a3));
                 x4=x3+h;
                 f4=f(x4);
                 x1=x2, x2=x3, x3=x4;
                 f1=f2, f2=f3,
                                  f3=f4 ;
               } while(fabs(h)>err);
               printf("\n Solution converges in iterations=%d",itr);
               printf("\n\n Root =%f",x4);
               getch();
                }
```



#### LAB ASSIGNMENTS : MULLER'S METHOD

**1.** Write C program to find the root of the equation  $x^3 - x - 4 = 0$  by Muller method correct to 4 decimal places.

Hint : Define f(x) = x \* x \* x - x - 4Input :  $x_1 = 0, x_2 = 1, x_3 = 2$ err = 0.00005

Output

**2.** Write a C program to find root of the equation  $3x + \sin x - e^x$  by Muller's method.

Hint : Define  $f(x) = 3 * x - \sin(x) - \exp(x)$ Input :  $x_1 = 0.5, x_2 = 1.0, x_3 = 0.0$ Output : Root = 0.36042

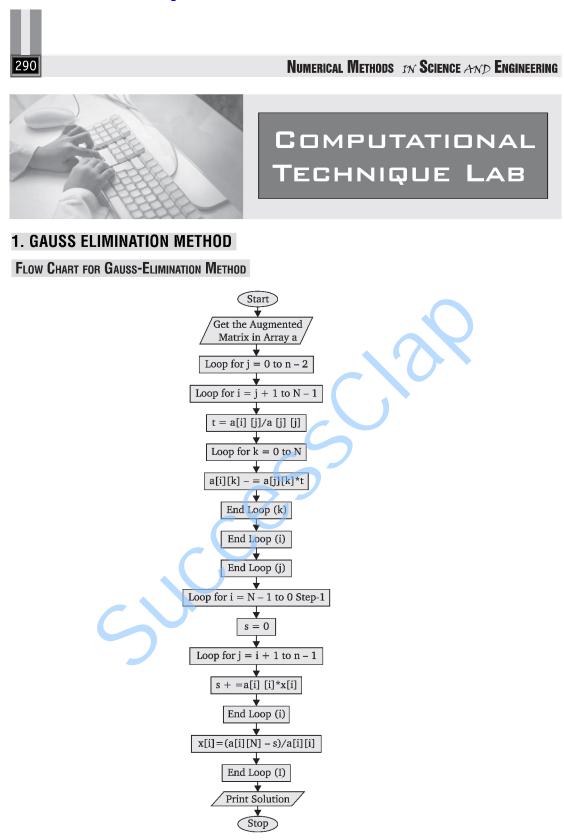
Root = 1.7963

**3.** Write C program to find the root of the equation

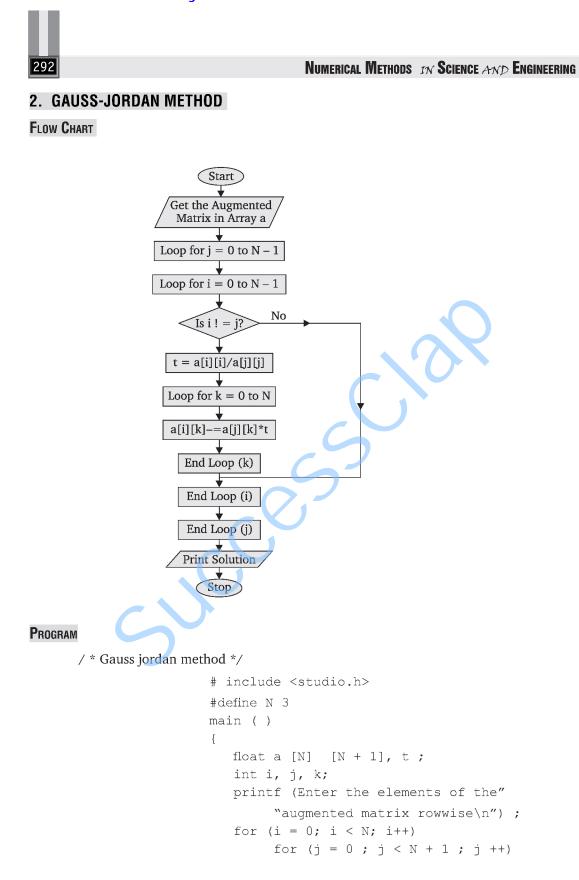
 $f(x) = x^{3} + 2x^{2} + 10x - 20 = 0$  by Muller method. Hint : Define f(x) = x \* x \* x + 2 \* x \* + 10 \* x - 20Input :  $x_{1} = 0, x_{2} = 1, x_{3} = 2,$  err = 0.0005

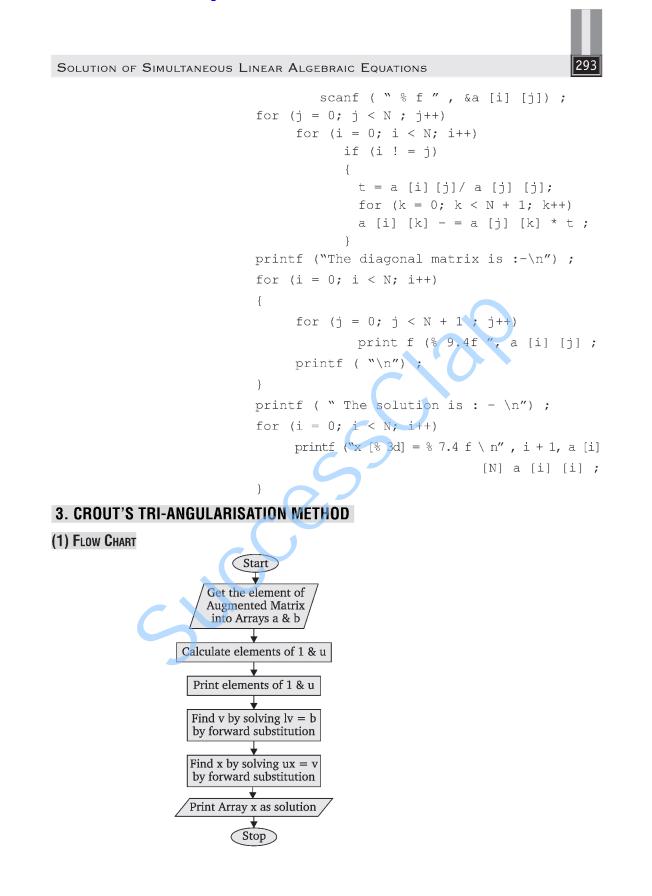
**Output :** Root = 1.3688

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```
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SOLUTION OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS
PROGRAM
     /* Gauss elimination method */
               # include <studio .h>
               # define N 4
               main ( )
               {
                          float a [N] [N + 1], x [N], t, s;
                          int i, j, k;
                          printf ("Enter the elements of the"
                               augmented matrix rowwise/n");
                          for (i = 0; i < N; i++)
                               for (j = 0; j < N + 1; J++)
                               scanf (" %", &a [i] [j]);
                          for (j = 0; j < N - 1; j++)
                               for i = j + 1; i < N; i++)
                          {
                                         🗧 a [i] 🔰 / a [j] [j] ;
                                       t
                          for (k = 0; k < n + 1; k++)
                                       a[i][k] - = a[j][k] * t;
                                  }
                          printf ("The upper triangular matrix"
                                 is :- n'';
                          for (i = 0; i < N; j++)
               {
                          for (j = 0; j < N; i++)
                                 printf (" %8 . 4f ", a [i] [j] ;
                          printf (" \ n ");
               }
               for (i = N - 1; i > = 0; i - -)
                          s = 0;
                          for (j = i + 1; j < N; j++)
                              s + = a [i] [j] *x [j];
                          x [i] = (a [i] (N) - S) / a [i] [i] ;
               printf ( "The solution is : - n'');
                          for (i = 0; i < N; i++)
                               printf ("x [\% 3d] = \% 7.4f\n", i + 1,
          x [i]);
                          }
```





```
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                                    NUMERICAL METHODS IN SCIENCE AND ENGINEERING
PROGRAM
                /* Crout triangularization method */
                        # include <studio .h>
                        # define N 4
                        typedef float matrix [N] [N] ;
                       matrix 1, u, a ;
                       float b [N], x [N], v [N];
                        void urow (int i)
                        {
                             float s ;
                             int j, k ;
                             for (j = 1; j < N; j++)
                             {
                             s = 0;
                             for (k = 0; k < N - 1;
                                                        k++)
                           s + = u [k] [j] * 1 [i] [k];
                             u [i] [j] = a [i] [j] - s;
                        }
                }
                void lcol (int j)
                {
                     float s ;
                     int i, k ;
                     for (i = j + 1)
                                         < N; i++)
                {
                             s = 0 ;
                     for (k = 0; k < = j - 1; k++)
                          s + = u [k] [j] * 1 [i] [k];
                     1 [i] [j] = (a [i] [j] - s) / u [j] [j] ;
                      }
                void printmat (matrix x)
                     int i, j ;
                     for (i = 0; i < N; i ++)
                     for (j = 0; j < N; j++)
                     printf ( " % 8 . 4f", x [i] [j] ;
                     printf ("\n") ;
                }
                }
                main ( )
                {
                int i, j, m;
                float s;
                printf ("Enter the elements of augmented"
                                   "matrix rowwise \n") ;
```

> 295 SOLUTION OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS (i = 0; i < N; i++)for { for (j = 0; j < N; j++)scanf ("% f", & a [i] [j]); scanf ("% f", & b [i]); } 1 and u \*/ for (i = 0; i < N;)i++) 1 [i] [i] = 1 . 0;for (m = 0; m < N; m++){ uprow (m); if (m < N - 1) lcol (m); } printf ("\t\tU\n"); printmat (u); printf ("\t\tL\n") ; printmat (l); for (i = 0; i < N; i++){ s = 0;for (j = 0; j < = i - 1; j++)s + = 1 [i] [j] \* v [j]; v [i] = b [i] - s; 7 for (i = N - 1; i > = 0){ s = 0 ; for (j = i + 1; j < N; j++)s + = u [i] [j] \* x [j] ; x [i] = (v [i] s/u [i] [i] ; } printf , ( "The solution is : - n'') ; for (i = 0; i < N; i++)printf (" X [% 3d] = % 6 . 4f/n" , i + 1, x [i]) ; 4. /\* LU DECOMPOSITION METHOD \*/ # include <conio .h> # include <studio. h> # include <math .h.> main ( ) { float a [15] [15] a1 [15] [15], b [15], X [15], au [15] [15], z [15]; float sum, t, big, ab, p ; int n, m, 1i, 1j, k, j, i, 1k, 13, m2, jj, kpl, kk, l; clrscr ( ); printf ("enter the value of n/n''); scanf (" % d", & n) ; printf ("enter the matrix row wise /n") ; for (i = 1; j < = n; i++)

```
for (j = 1; j < = n; j++)
```

scanf ("% f ", & a [i] [j] );

```
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```

## NUMERICAL METHODS IN SCIENCE AND ENGINEERING

```
printf ("enter the matrix B n'');
       for (j = 1; j < = n; j++)
       scanf ( " % f" , & b [j] ) ;
for (i = 1; i < = n; i++)
for (j = 1; j < = n; j++)
{
       a1 [i] [j] = 0;
       au [i] [j] = 0;
}
for (i = 1; i < = n; i++)
{
         au [i] [i] = 1 ;
         a1 [i] [1] = a [i] [1] ;
       au [1] [i] = a [1] [i] / a1 [1]
                                        [1];
}
for (j = 2; j < = n; j++)
for (i = j; i < = n; i++)
{
       sum = 0;
            for (k = 1; k \le j - 1; k++)
            sum=sum+a1 [i] [k] * au [k] [j] ;
            a1 [i] [j] = a [i] [j] - sum ;
}
if ( j ! =n)
(
       for (jj = j + 1; jj < = n; jj++)
            sum=0;
            for (kk = 1; kk < j - 1; jj++)
            sum=sum + a1 [j] [kk] * au [kk] [jj];
           au [j] [jj] = (a [j][jj] - sum) / a1 [j][j];
              }
                   }
z [1] = b [1] / a1 [1] [1];
for (i = 2; i < = n; i++)
{
       sum=0;
       for (k = 1; k < i - 1; k++)
       sum=sum+a1 [i] [k] * z [k];
       z [i] = (b [i] - sum)/a1 [i] [i];
}
```

```
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SOLUTION OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS
                       x [n] = z [n] ;
                       for (i = 2; i < = n; i++)
                          1 = n - i + 1;
                               sum=0;
                          for (k = 1 + 1; k < = n; k++)
                              sum= sum+au [1] [k] * x [k];
                          x [1] = z [1] - sum;
          }
               printf ("\n") ;
               printf ( "lower triangular matrix n'');
                for (i = 1; i < = n; i++)
                {
                       for (j = 1; j < = n; j++)
                       printf ("% 5. 2f ", a1 [i] [j])
                       printf ("\n'');
                }
               printf ( "upper triangular matrix \n") ;
                for (i = 1; i < = n; i++)
                {
                       for (j = 1; j < = n; j++)
                       printf ( ``{5.3f", au [i] [j]);
                       printf ( \\n'')
                                       );
                }
               printf ( "solution vectorn'');
               for (i = 1; i < = n; i++)
               printf ( "% 5. 2f " , x [i] ) ;
               printf ( ``\n'');
               getch 🔪 )];
5. SOLUTION OF SYSTEM OF EQUATIONS USING GAUSS SEIDAL METHOD
         */
              // include <conio .h>
                # include <studio .h>
               # include <math .h>
               main ( )
                {
                       float a [15] [15] , b [15] , x [15] , oldx [15], eps,
          c , big, sum;
                       int n, niter1, i, j, k, l, ii ;
                       clrscr ( ),
                       printf ( "enter the value of N, NITHER, ESPn'');
                       scanf ( "% d% d% f" , &n, &niteral, &eps) ;
```

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}

## NUMERICAL METHODS IN SCIENCE AND ENGINEERING printf ("enter the matrix A n''); for (i = 1; i < = n; i++)for (j = 1; j < = n; j++)scanf ``%f", a[i][j]; scanf ("enter the array B n''); for (i = 1; i < = n; i++)scanf ( " % f, & b [i] ; printf ("enter the array 01dx1n" for (i = 1; i < = n; i ++); for (j = 1; j < = n, j + 1)( "%f ", a [i], [j]); printf print ( " % / n") ; printf (" the array B/n''); for (i = 1, i < = n; i ++){ x (i) = 0/dx [i] printf ( "% f " , b [i]); } printf ("\n"); for (i = 1; i < = n; i ++);for (i = 1; i= n iter ; i ++) { sum = 0 for (j = 1 : j < = n ; j + j) if([i - j]; = 0)sum = sum + a [i] [j] \*x [j] ; x [i] = b [i] - sum / a [i] [i];} printf (n iter = % d, i) ; for (i = 1 ; i < = n; i ++)</pre> printf (" % 12.6 f" , x [i]) ; printf ("\n") ; big = abb (x [i] - old x [i]);for (k = 2; k < = n, k++){ c = abb (xck) - old x [k];if (big < = c)big = c;For (i = 1; i < = n; i++)old x [i] = x [i] ;

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FINITE DIFFERENCES AND INTERPOLATION

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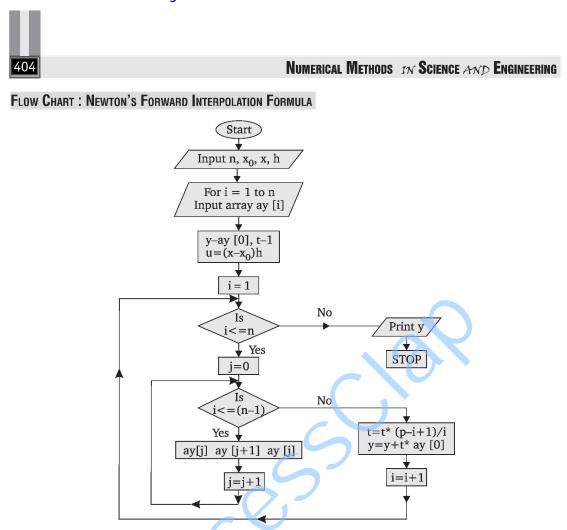
# **1. NEWTON'S FORWARD INTERPOLATION FORMULA**

#### Symbols Used

- $x_0$  = initial value of x
- h =length of interval
- n = number of subintervals
- x = value of x at which we have to find the value of y
- y = value of y at x
- ay = array to store the different values of y
- t = temporary variable

### **ALGORITHM : NEWTON'S FORWARD INTERPOLATION FORMULA**

- Step 1. Start
- **Step 2.** Input n, x<sub>0</sub>, h, x
- **Step 3.** Input values of av
- **Step 4.** y = ay [0], t = 1
- **Step 5.**  $u = (x-x_0)/h$
- **Step 6.** For i = 1 to n
- **Step 7.** For j = 0 to (n-i)
- **Step 8.** y[j] = y[j+1] y[j]
- **Step 9.** End of j loop.
- **Step 10.** t = t \* (p i + 1)/i
- **Step 11.** y = y + t \* ay [0]
- Step 12. End of i loop
- Step 13. Print y
- Step 14. STOP



```
PROGRAM.
          Following is a program to show the Newton's forward interpolation formula.
          NEWTON'S FORWARD INTERPOLATION FORMULA
                                                             (UPTUMCA-2004)
           # include<studio.h>
           #include<conio.h>
            void main ( )
                {
                 clrscr ( );
                float ay [30], x0, h, x, y, t = 1, u;
                 int n, i, j;
                printf ("Enter the value of n \ );
                scanf ("% d", & n);
                printf ("Enter the initial value of x n'');
                 scanf (``%d", &x0);
                printf ("\n enter length of each interval\n");
                 scanf ("%f", &h);
                 for (i = 0; i < n; i++)
```

```
405
FINITE DIFFERENCES AND INTERPOLATION
                       print f ("Enter the value of y (%d) = " , i);
                {
                       scan f ( "%f", & ay [i]);
                }
               printf("Enter the value of x for which value of y is wanted n'');
                scanf (``%f", &x)';
               y=ay [0];
               u = (x - x0) / h;
                for (i=1; i<=n; i++)</pre>
                        {
                                 for (j=0; j<=n-1; j++)
                                 ay [j] = ay [j+1]-ay [j];
                                 t=t* (u-i+1)/i;
                                 y=y+t*ay [0]
                        }
               printf ("\n Value of y at x=%. 2 fis is \% .2f ", x, y); getch ();
                           }
          Output : NEWTON'S FORWARD INERPOLATION FORMULA
          Enter the value of n
          6
          Enter the initial value of x
          Ω
          enter length of each interval
          1
          Enter the value of
                                  y (0) =
                                                 1
          Enter the value of
                                  y(1) =
                                                 3
          Enter the value of
                                  y (2) =
                                                 11
          Enter the value of
                                 y (3) =
                                                 31
          Enter the value of
                                  y (4) =
                                                 69
          Enter the value of
                                                 131
                                  y (5) =
          Enter the value of
                                  y (6) =
                                                 223
          Enter the value of x for which value of y is wanted
          3.4
          Value of y at x = 3.40 is 43.70
```

#### LAB ASSIGNMENT : NEWTON'S FORWARD INTERPOLATION FORMULA :

**1.** Write a C program for the Newton's forward interpolation formula to find the value of *y* at x = 2.7 from the following data

x	1	2	3	4	5	6	7	8
<i>f</i> ( <i>x</i> )	1	8	27	64	125	216	343	512



**Hint: Input**  $x_0 = 1, h = 1, n = 7$ 

**Output** f(2.7) = 50.65

**2.** Write a C program to find f (3.4) using the following values by Newton's forward interpolation formula

x	0	1	2	3	4	5	6
f(x)	1	3	11	31	69	131	223

**Hint: Input**  $x_0 = 0, h = 1, n = 6$ **Output** f(3.4) = 43.704

### 2. NEWTON'S BACKWARD INTERPOLATION FORMULA

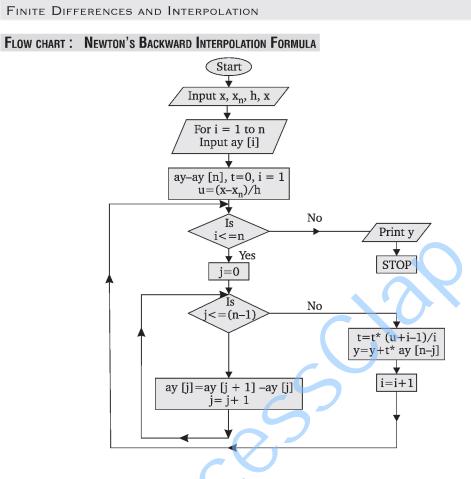
#### Symbols Used

- n = number of sub-intervals
- h = length of interval
- $x_n = last value of x_i$
- $\mathbf{x} =$ values of x at which we have to find the value of y
- a [y] = an array to store different values of y
  - y = value of y at x

# Algorithm : Newton's Backward Interpolation Formula

Step 1:	Start
Step 2:	Input n, x <sub>n</sub> , h, x
Step 3 :	For $i = 0$ to n
Step 4 :	Input ay [i]
Step 5 :	ay = ay [n], t = 1
Step 6 :	$u = (x - x_n)/h$
Step 7:	For $i = 1$ to n
Step 8 :	For $j = 0$ to $(n-i)$
Step 9:	ay [j] = ay [j+1] – ay [j]
Step 10 :	End of j loop
Step 11 :	$t = t^* (u+i-1)/i$
Step 12 :	$y = y + t^* ay [n - i]$
Step 13 :	End of j loop
Step 14 :	Print y
Step 15 :	STOP.

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PROGRAM : Following is a program to demonstrate the Newton's Backward Interpolation Formula. NEWTON'S BACKWARD INTERPOLATION FORMULA

```
#include<studio.h>
#include<conio .h>
void main ()
{
    clrscr ();
    float ay [30], xn, h, x, y, t=1, u;
    int n, i, j;
    printf ("Enter the value of n\n");
    scanf ("%d", &n);
    printf ("Enter the last value of x\n");
    scanf ("%f ", & xn);
    printf ("\n enter length of each interval\n");
    scanf ("%f ", &h);
    for (i=0; i<=n; i++)</pre>
```

```
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                                    NUMERICAL METHODS IN SCIENCE AND ENGINEERING
                       print f ("Enter the value of y(%d) = ", i);
                {
                       scanf ("% f ", & ay [i]) ;
               }
          printf ("\n Enter the value of x for which value of y is
          watned\n");
               scanf (``%f", &x) ;
               y=ay [n] ;
               u=(x-xn)/h;
               for (i=1; i = n; i++)
                   {
                         for (j=0; j<=n-i; j++)
                       ay [j]=ay[j+1]-ay [j];
                         t =t* (u+i-1)/i;
                         y=y+t*ay [n-i];
          printf ("\n value of y at x = % .2f
                                               is%.2f"
                                                             V)
                                                                ;
                                                                   getch
                                                         x,
          ();
          Output : NEWTON'S BACKWARD INTERPOLATION FORMULA
          Enter the value of n
          4
          Enter the last value of x
          60
             Enter length of each interval
          10
          Enter the value of
                               y (0)
                                               42
                                        =
          Enter the value of
                                               87
                               y (1)
                                        =
          Enter the value of
                                y (2)
                                        =
                                               126
          Enter the value of
                                y (3)
                                               174
                                        =
          Enter the value of
                                y (4)
                                               193
                                       =
             Enter the value of x for which value of y is wanted
          44
          value of y at x=44.00 is 145.06
```

#### LAB ASSIGNMENT : NEWTON'S BACKWARD INTERPOLATION FORMULA

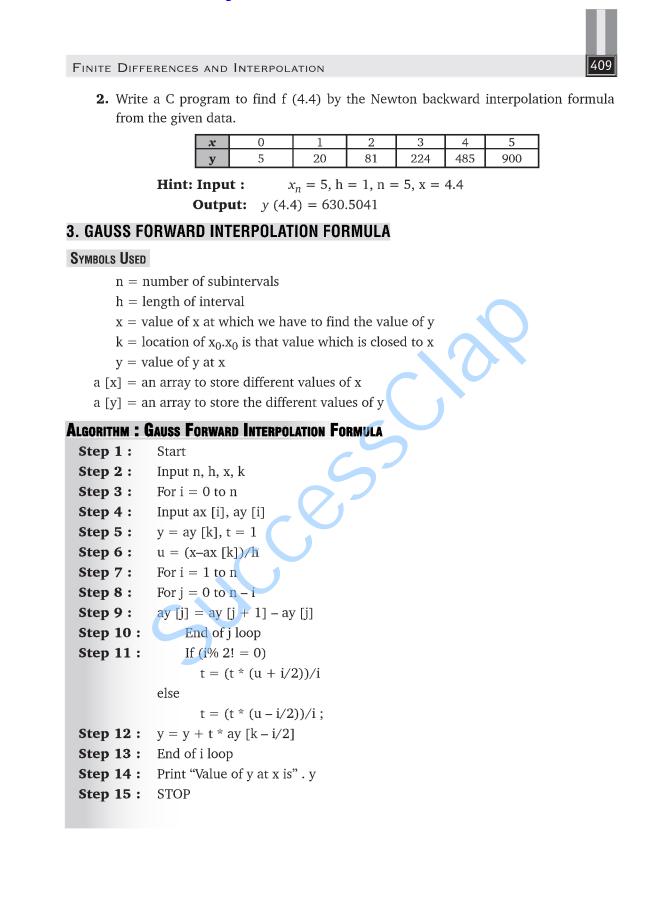
**1.** Write a C program for Newton's Backward Interpolation formula to find the value of f (7.5) for the data given below :

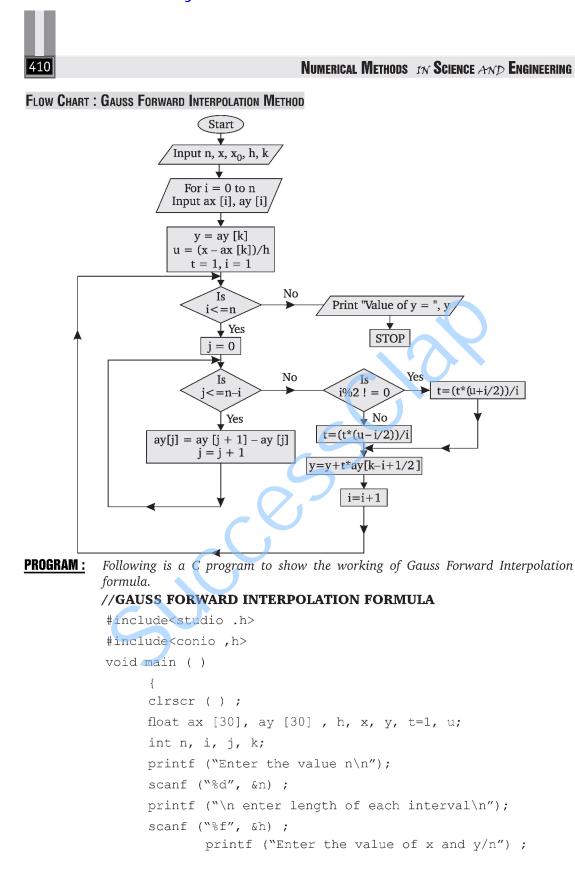
	x	1	2	3	4	5	6	7	8
<b>f(x)</b> 1 8 27 64 125 216 34	t(r)	1	8	27	64		26	343	512

Hint: Input

 $x_n = 5, h = 1, n = 7. x = 7.5$ 

**Output** f(7.5) = 421.875





```
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FINITE DIFFERENCES AND INTERPOLATION
               for (i=0; i<=n; i++)</pre>
                       scanf ("%f %f ", & ax [i], & ay [i] ;
               printf ("\n Enter the value of x for which value of y
         is wanted\n");
               scanf ("%f", &x);
               printf ("\n enter the location of x0 i.e. k/n'');
               scanf (``%d", &k);
               y=ay [k];
               u=(x-ax [k]) /h;
               for (i=1; i<=n; i++)</pre>
                  {
                       for (j=0; j<=n-i; j++)
                         ay [j]=ay[j+1]-ay [j];
                       if (i%2 ! = 0)
                               t=(t*(u+i/2))/i;
                      else
                                t = (t * (u - i/2)) / i;
                       y=y+t*ay[k-i/2];
                            }
                      printf ("\n value of y at x=%. 2f is % .2f ", x,
         y); getch ();
         Output : GAUSS FORWARD INTERPOLATION FORMULA
         Enter the value of n
         5
            enter length of each interval
          .5
         Enter the value of x and y
         2.5 24.145
         3 22.043
         3.5 20.225
         4
              18.644
         4.5 17.262
         5
             16.047
         Enter the value of x for which value of y is wanted
         3.75
           enter the location of x0 i.e. k
         2
         value of y at x=3.75 is 19.41
```



#### NUMERICAL METHODS IN SCIENCE AND ENGINEERING

#### LAB ASSIGNMENT : GAUSS FORWARD INTERPOLATION FORMULA

1. Write a C program to find value of y (30) by Gauss Forward Interpolation formula from the data given below :

x	21	25	29	33	37
f(x)	18.4708	17.8144	17.1070	16.3432	15.5154

**Hint : Input :** n = 4, h = 4, x = 30, k = 2

**Output :** Value of y = 16.92

2. Write a C program for Gauss Forward interpolation formula to find value of f (2.3) from the given data

		x	1	2	3	4	5
		f(x)	1	-1	1	-1	1
Hint:	Input :	n = 4	4, h = 1, x	= 2.3, k	= 1		

**Output:** Value of y = -0.146600.

# 4. GAUSS BACKWARD INTERPOLATION FORMULA

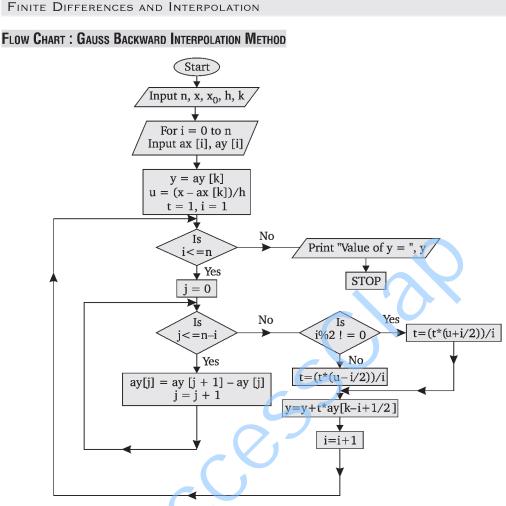
#### SYMBOLS USED

(Note : same as used in Gauss forward Interpolation formula).

#### **ALGORITHM: GAUSS BACKWARD INTERPOLATION FORMULA**

```
Step 1:
            Start
Step 2 :
            Input x, h, x<sub>0</sub>, k
Step 3 :
            For i = 0 to n
Step 4 :
            Input ax [i], ay [i]
            y = ay [k], t = 1
Step 5 :
            u = (x - ax [k])/h
Step 6 :
           For i = 1 to n
Step 7 :
            For j = 0 to n - i
Step 8 :
Step 9 :
            ay[j] = ay[j + 1] - ay[j]
Step 10 : End of j loop
Step 11 : if (1\%2! = 0)
                      t = (t * (u - i/2))/i
            else
                      t = (t * (u + i/2))/i
Step 12 : y = y + t * ay [k - (i + 1)/2]
Step 13 : End of i loop.
Step 14 : Print "value of y=", y
Step 15 :
            Stop.
```

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**PROGRAM :** Following is a C program to show the working of Gauss Backward Interpolation Method.

```
GAUSS BACKWARD INTERPOLATION FORMULA
```

```
#include<studio .h>
#include<conio .h>
void main ()
      {
     clrscr ( ) ;
            ax [30], ay [30], h, x, y, t = 1, u;
     float
     int n, i, j, k;
     printf ("Enter the value of n \mid n'');
     scanf ("%d", &n) ;
     printf ("\n enter length of each interval\n") ;
     scanf ("%f", &h) ;
             printf ("Enter the value of x and y\n ") ;
     for (i=0; i<=n;i++)</pre>
      {
             scanf ("%f %f", &ax [i], &ay [i]);
```

```
414
                                    NUMERICAL METHODS IN SCIENCE AND ENGINEERING
                printf ("\n Enter the value of x for which value of y
          is wanted\n") ;
                scanf (``%f", &x) ;
               printf ("\n enter the location of x0 i.e., k n'');
                scanf ("%d", &k) ;
                y=ay [k] ;
                u=(x-ax [k])/h;
                for (i=1; i \le n; i++)
                   ł
                         for (j=0; j<=n-i; j++)</pre>
                       ay [j] = ay [j+1] - ay [j];
                          if (i % 2 ! = 0)
                                t=(t*(u-i/2))/i;
                       else
                                t=(t*(u+i/2))/i;
                       y=y+t*ay [k-(i+1)/2];
                printf ("\n Value of y at x=%.2f is
                                                      %.2f", x, y) ;
                getch ();
          Output : GAUSS BACKWARD INTERPOLATION FORMULA
          Enter the value of n
          3
             enter length of each interval
          1
          Enter the value of x and y
          4
                270
          5
                648
          6
                1330
          7
                2448
            Enter the value of x for which value of y is wanted
          5.8
            enter the location of x0 i.e. k
          1
          Value of y at x=5.80 is 1169.28
```

LAB ASSIGNMENT : GAUSS BACKWARD INTERPOLATION METHOD

**1.** Write a C program for Gauss Backward interpolation method to find y (1.15) form data given below :

	x	1	1.10	1.20	1.30
	У	1.0	1.04881	1.09544	1.14017
Hint :	Input :	n = 3, h =	= 0.10, k =	1, x = 1.15	5

```
Output : Value of y = 1.072397
```

**2.** Write a C program for Gauss Backward Interpolation Method to find the value for 1936 from the given data.

x	1901	1911	1921	1931	1941	1951
f(x)	12	15	20	27	39	52

**Hint : Input :** n = 5, h = 10, x = 1936, k = 3

**output :** Value of y = 32.3437.

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FINITE DIFFERENCES AND INTERPOLATION

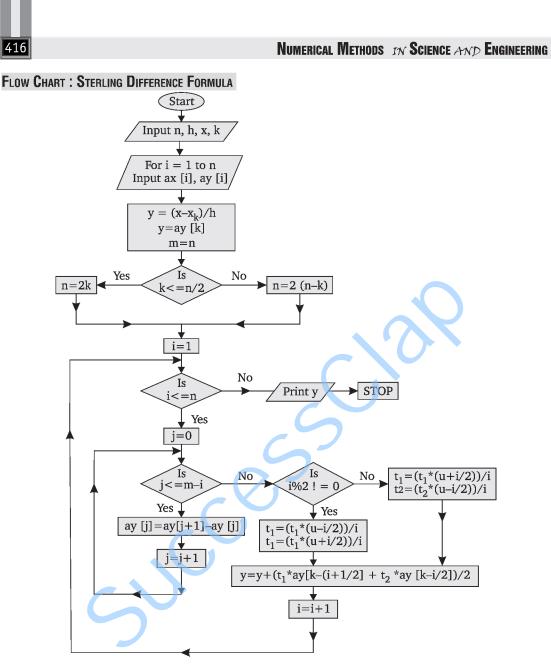
# **5. STIRLING'S DIFFERENCE FORMULA**

#### Symbols Used

- n = number of subintervals
- h = length of interval
- x = value of x at which we have to find the value of y
- $k = location of x_0. x_0$  is that value which is closed to x
- y = value of y at x
- a [x] = an array to store the different values of x
- a [y] = an array to store the different values of y

### **ALGORITHM : STERLING'S DIFFERENCE FORMULA**

Step 1 :	Start
<b>Step 2 :</b>	Input n, h
Step 3 :	For $i = 1$ to n
Step 4 :	Input ax [i], ay [i]
Step 5 :	Input x, k
Step 6 :	$u = (x - x_k)/h$
Step 7 :	y = ay [k], m = n
Step 8 :	if (k<=n/2)
	n = 2k
	else
	n = 2 (n - k)
Step 9:	For $i = 1$ to n
<b>Step 10 :</b>	For $j = 0$ to $(m - i)$
Step 11 :	ay[j] = ay[j + 1] - ay[j]
Step 12 :	End of j loop
Step 13 :	if $(i\%2! = 0)$
	$t_1 = (t_1 * (u - i/2))/i$
	$t_2 = (t_2 * (u + i/2))/i$
	else
	$t_1 = (t_1 * (u + i/2))/i$
	$t_2 = (t_2 * (u - i/2))/i$
Step 14 :	$y = y + (t_1 * ay [k - (i + 1)/2] + t_2 * ay [k - i/2])/2$
Step 15 :	End of i loop
Step 16 :	-
Step 17:	
	r



```
PROGRAM : Following is a C program showing the working of Stirling Difference Formula for interpolation.
```

```
//STIRLING'S INTERPOLATION FORMULA
```

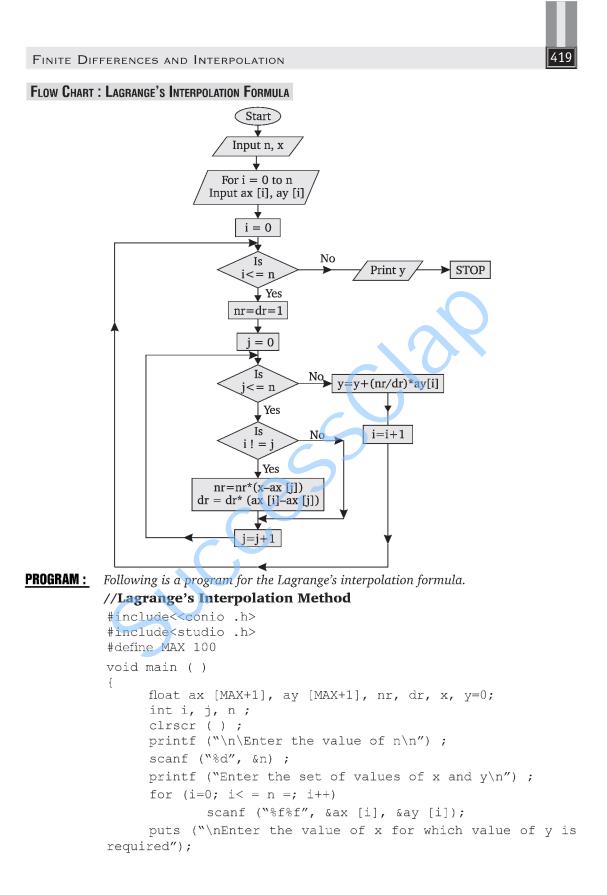
```
#include<studio.h>
#include<conio.h>
void main ()
        {
        clrscr ();
        float ax [30], ay [30], h, x, y, t1=1, t2=1, u;
        int n, i, j, m, k;
```

> 417 FINITE DIFFERENCES AND INTERPOLATION printf ("Enter the value of n n'); Scanf ("%f", & h); printf ("\n enter length of each interval\n"); scanf (``%f", &n); printf ("Enter the value of x and y\n ") ; for (i=0, i < = n; ++){ scanf ("%f %f ", &ax [i], &ay [i]); } printf ("Enter the value of x for which value of y is wanted\n") ; scan (``%f", &x) ; printf ("\n enter the location of x0 i.e. k\n") ; scanf (``%d", & k); printf ("\n enter the location of x0 i.e. k n'); scanf ("%d", &k) ; y=ay [k] ; u=(x-ax [k])/h;m=n; if (k < = n/2)n=2 k;else n = 2\* (n-k);for (i=1; i<=n; i++) { for (j=0; j<=m-i; j++) ay[j]=ay [j+1]-ay [j]; if  $(i \ge 2 ! = 0)$ t1=(t1\*(u-i/2)) /i; ł t2=(t2\* (u+i/2)) /i; elsé t1=(t1\*(u+i/2))/i; t2=(t2\*(u-i/2))/i;y=y+(t1\*ay[k-(i+1) / 2] + t2\*ay[k-i/2)) /2 ; print f ("\n Value of y at x=% .2f is % .2f ", x, y) ; getch ( ) ; **Output : STIRLING'S INTERPOLATION FORMULA** Enter the value of n enter length of each interval 5 Enter the value of x and y 10 492 15 483 20 472 459 25

NUMERICAL METHODS IN SCIENCE AND ENGINEERING 418 30 453 Enter the value of x for which value of y is wanted 19 enter the location of x0 i.e. k 2 Value of y at x=19.00 is 474.49 LAB ASSIGNMENT : STIRLING DIFFERENCE FORMULA **1.** Write a C program to find the value of y [28] from the given data using Stirling Difference Formula. 20 25 x 30 35 40 y 49225 48316 47236 45926 44306 **Hint : Input** : x = 28, n = 4, h = 5,  $x_0 = 30$ , k = 2**Output :** y(28) = 47692. **2.** Using Stirling difference formula, write a C program to find y (25) by given data : x 20 24 28 32 y 24 32 34 40 **Hint : Input :** h = 4, n = 3, x = 25, k = 1**Output :** y (25) = 32.945287 6. LAGRANGE'S INTERPOLATION FORMULA FOR UNEQUAL INTERVAL SYMBOLS USED n = number of subintervalsx = value of x at which we have to find the value of y nr = numerator of each term of Lagrange's formula dr = denominator of each term of Lagrange's formula y = value of y at xa [x] = an array to store different values of x a [y] = an array to store the difference values of y' **ALGORITHM : LAGRANGE'S INTERPOLATION FORMULA** Step 1: Start **Step 2**: Input n, x Step 3: For i = 0 to n Step 4 : Input ax [i], ay [i] Step 5 : For i = 0 to n Step 6 : nr=dr=1For j = 0 to n **Step 7 :** Step 8: if (i ! = j)nr = nr \* (x - ax [j])dr = dr \* (ax [i] - ax [j])Step 9 : End of j loop. **Step 10 :** y = y + (nr/dr) \* ay [i]End of i loop **Step 11 :** 

Step 12 : Print y

Step 13 : Stop.



#### NUMERICAL METHODS IN SCIENCE AND ENGINEERING 420 scanf (``%f", &x); for (i=0; i<=n;'i++)</pre> { nr=dr=1; for (j=0; j<= n; j++)</pre> if (i ! = j){ nr\*=x-ax [j]; dr\*=ax [i] -ax[j] ; y=(nr/dr) \*ay [i] ; printf ("\n when x=%f, y=%f", x, y) ; getch (); **OUTPUT : LAGRANGE'S INTERPOLATION METHOD** Enter the value of n 4 Enter the set of values of x and y 1 8 2 15 4 19 8 32 10 40 Enter the value of x for which value of y is required when x = 5, y = 22.7460LAB ASSIGNMENT : LAGRANGE'S INTERPOLATION FORMULA 1. Write a C program using Lagrange's formula to find f (4) by the given data 0 2 5 x 7 f(x)2 5 8 **Hint :** Input : n=3, x=4**Output :** f (4)=8.4

**2.** Write a C program to find y (5) using Langrange's interpolation formula for the data given below :

x	1	3	4	8	10
f(x)	8	15	19	32	40

**Hint : Input :** n = 4, x=5

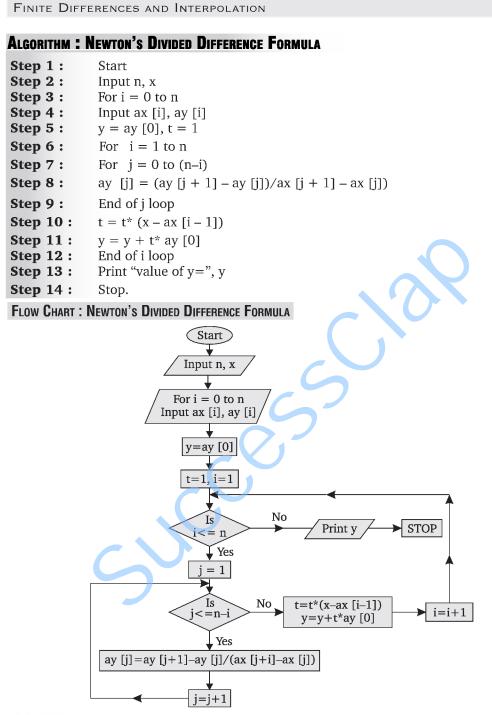
**Output :** y (5) = 11.318

### 7. NEWTON'S DIVIDED DIFFERENCE FORMULA

#### SYMBOLS USED :

- n = number of subintervals
- x = value of x at which we have to find the value of y
- y = value of y at x
- a [x] = an array to store the different values of x
- a [y] = an array to store the different values of y.

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**PROGRAM :** Flowing program shows the working of Newton's Divided Difference formula for interpolation. //NEWTON'S DIVIDED DIFFERENCE INTERPOLATION FORMULA

#include<studio .h>
#include<conio .h>

NUMERICAL METHODS IN SCIENCE AND ENGINEERING

```
void main ()
               clrscr ( ) ;
               float ax [30], ay [30], x, y, t=1;
               printf ("Enter the value of n n');
               scanf (``%d", &n) ;
               printf ("\n enter value of x n'');
               scanf (" %f ", &x) ;
               printf ("Enter the value of x and y n ");
               for (i=0; i<=n;i++)</pre>
                scanf ("%f %f", &ax [i], &ay [i]);
               }
               y=ay [0] ;
               for (i=1; i<=n; i++)
                  {
                for (j=0;j<=n;j++)</pre>
                       ay [j]=(ay [j+1] - ay [j]/ (ax [j+1] - ax [j]) ;
                t = t^{*} (x-ax [i-1]);
                y=y+t*ay [0] ;
               printf ("\n Value of y at x=% .2f is % .2f ", x, y) ;
               getch ();
          Output : NEWTON'S DIVIDED DIFFERENCE INTERPOLATION FORMULA
          Enter the value of n
          enter value of x
          1.6
          Enter the value of x and y
                3.49
          1
          1.4
                4.82
          1.8
                5.96
                6.5
          2.2
          Value of y at x=1.60 is 5.44
LAB ASSIGNMENT: NEWTON'S DIVIDED DIFFERENCE FORMULA
```

**1.** Write a computer program in C language to find value of f (15) by using Newtons divided difference formula from the following table :

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028
Hint	t: Input:	n = 5, x = 15				

**Output :** Value of y at x = 15 is 3150.

**2.** Write a C program for the Newton's divided difference formula to find value of f(10) from the following data :

x	5	6	9	11
f(x)	12	13	14	16
Lint . In	$p_{11t} \cdot p = $	2 m - 10		

**Hint :** Input : n = 3, x = 10

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**Output :** Value of *y* at x = 10 = 14.667.

 $\mathbf{x}\mathbf{x}\mathbf{x}\mathbf{x}\mathbf{x}\mathbf{x}\mathbf{x}\mathbf{x}$ 

NUMERICAL DIFFERENTIATION AND INTEGRATIONS

Image: Computation and Integration

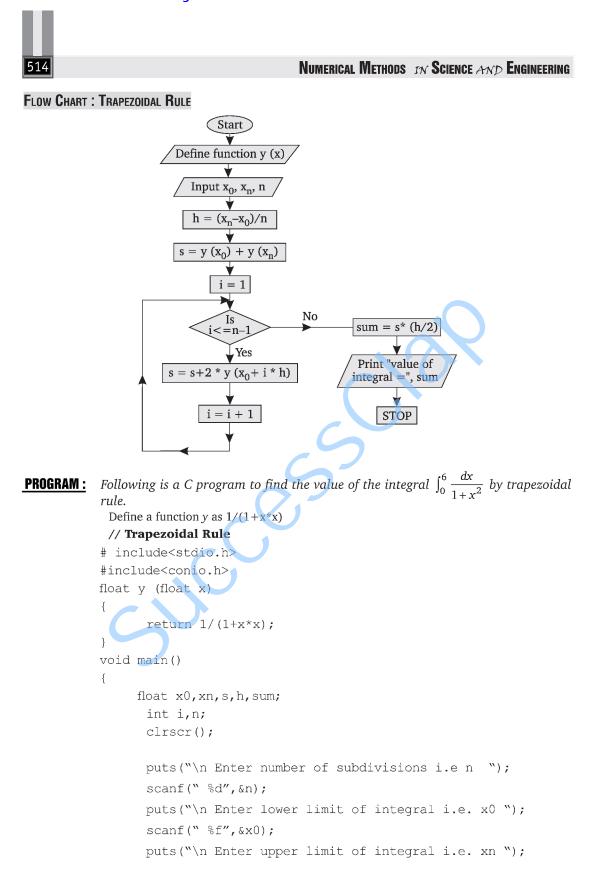
Image: Computation and Integration and Integration

Image: Computation and Integration and Integratio

- $x_0 =$ lower limit of integral
- $x_n$  = upper limit of integral
- h =length of subinterval
- y(x) = function to be integrated

# **ALGORITHM : TRAPEZOIDAL RULE**

Step 1 :	Start
Step 2 :	Input x <sub>0</sub> , x <sub>n</sub> , n
Step 3 :	$h = (x_n - x_0)/n$
Step 4 :	$s = y(x_0) + y(xn)$
Step 5 :	For $i = 1$ to $n - 1$
Step 6 :	$s = s + 2 * y (x_0 + i * h)$
Step 7 :	End of i loop
Step 8 :	Sum = s * (h/2)
Step 9 :	Print "Value of integral = ", sum
Step 10 :	Stop.



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```
NUMERICAL DIFFERENTIATION AND INTEGRATIONS
                    scanf(" %f",&xn);
            h=(xn-x0)/n;
                    s=y(x0)+y(xn);
                   for(i=1;i<=n-1;i++)</pre>
                       s + = 2 * y (x 0 + i * h);
                    sum=s*(h/2);
                    printf("\n Value of Integral is %.3lf\n",sum);
                    getch();
            }
            Output: TRAPEZOIDAL RULE
            enter number of subdivisions i.e. n
            6
            enter lower limit of integral i.e. x0
            enter upper limit of integral i.e.
            Value of Integral is 1.411
LAB ASSIGNMENT : TRAPEZOIDAL RULE
   1. Write a C program to solve the following integral by trapezoidal rule
                        \int_{0.2}^{1.4} (\sin x - \log_e x + e^x) dx
      Hint : Define a function y
              sin (x - loge (x) + exp (x))
      Input : x_0 = 0.2 x_n = 1.4 n = 12
      Output : Value of integral = 4.056174
  2. Write a C program to calculate the value of integral \int_{4}^{5.2} \log x \, dx by trapezoidal rule.
      Hint : Define function y = \log(x)
      Input : x_0 = 4, x_n = 5.2 n = 6
      Output : Value of integral = 1.827648
```

### 2. SIMPSON'S 1/3 RULE

#### SYMBOLS USED

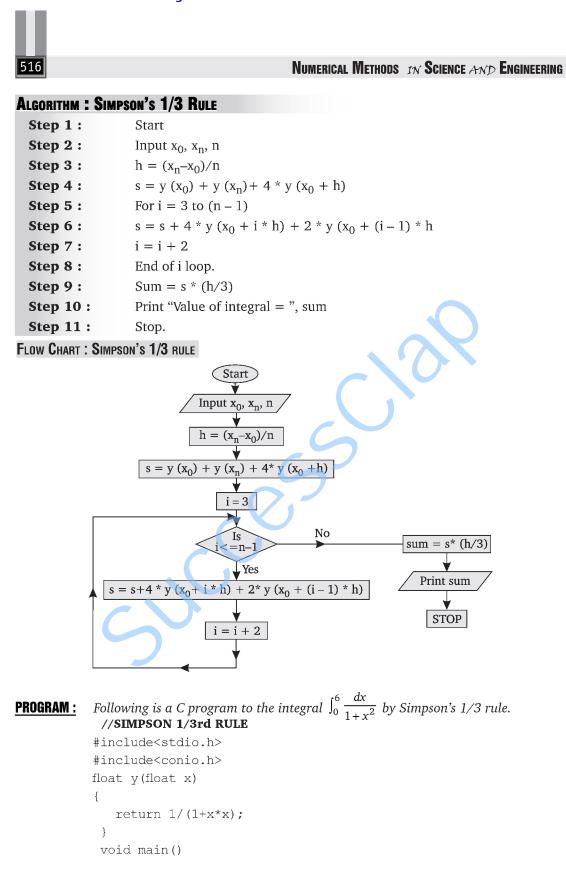
 $x_0 =$ lower limit of integral

 $x_n =$  upper limit of integral

n = no. of subdivisions

h = length of subinterval

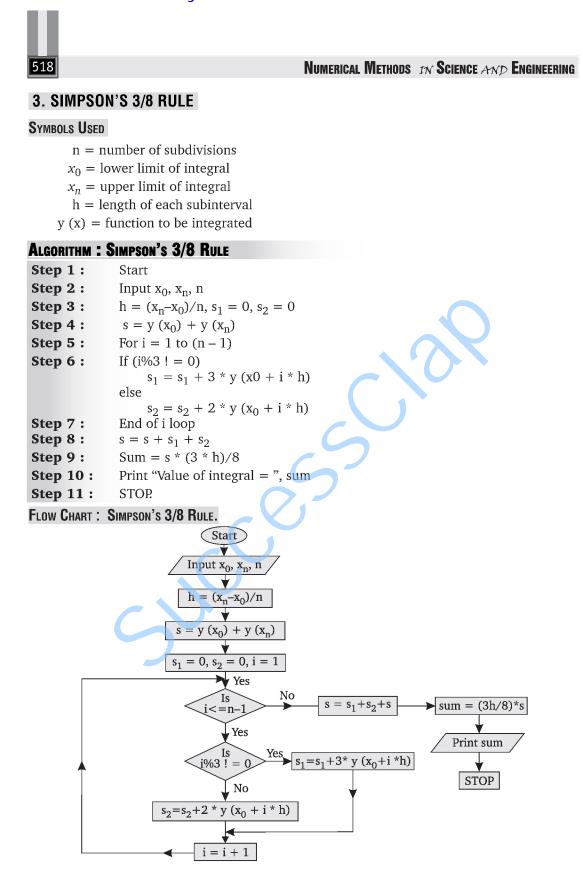
y(x) = function to be integrated



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```
NUMERICAL DIFFERENTIATION AND INTEGRATIONS
           {
                 float x0, xn, h, s, sum;
                 int i,n;
                 clrscr();
                 puts("\n enter number of subdivisions i.e. n ");
                 scanf("%d",&n);
                 puts("\n enter lower limit of integral i.e. x0 ");
                 scanf("%f",&x0);
                 puts("\n enter upper limit of integral i.e. xn ");
                 scanf("%f",&xn);
                 h=(xn-x0)/n;
                 s=y(x0)+y(xn)+4*y(x0+h);
                 for(i=3;i<=n-1;i+=2)</pre>
                     s = 4 * y (x0 + i * h) + 2 * y (x0 + (i-1) * h);
                 sum=s*(h/3);
                 printf("\n Value of Integral is %.31f\n", sum);
                 qetch();
          }
          Output: SIMPSON 1/3 rd RULE
          enter number of subdivisions i.e. n
          enter lower limit of integral i.e. x0
          enter upper limit of integral i.e. xn
          6
          Value of Integral is 1.366
LAB ASSIGNMENT : SIMPSON'S 1/3 RULE
```

- **1.** Write a C program to solve the integral  $\int_0^4 e^x dx$  using Simpson's 1/3 rule. **Hint**: Define function  $y = \exp(x)$  **Input**:  $x_0 = 0$ ,  $x_n = 4$ , n = 4**Output**: Value of integral = 53.87
- **2.** Write a C program to solve the integral  $\int_0^7 \frac{1}{x} dx$  using Simpson's 1/3 rule. **Hint**: Define function y = (1/x) **Input**:  $x_0 = 1, x_n = 7, n = 6$ **Output**: Value of integral = 1.9587
- **3.** Write a C program to solve the integral  $\int_{0.5}^{0.7} x^{1/2} e^{-x} dx$  by Simpson's 1/3 rule. **Hint**: Define function y = sqrt (x) \* exp (-x) **Input**: x<sub>0</sub> = 5, x<sub>n</sub> = 0.7, n = 4 **Output**: Value of integral = 0.08483



```
519
 NUMERICAL DIFFERENTIATION AND INTEGRATIONS
           Following is a C program to the integral \int_0^1 \frac{dx}{1+x} by Simpson's 3/8 rule. //SIMPSON 3/8th RULE
PROGRAM :
           #include<stdio.h>
           #include<conio.h>
           float y(float x)
               return 1/(1+x);
            }
            void main()
           {
                  float x0, xn, h, s, s1=0, s2=0, sum;
                  int i,n;
                  clrscr();
                  puts("\n enter number of subdivisions
                                                             i.e. n ");
                  scanf("%d",&n);
                  puts("\n enter lower limit of integral i.e. x0 ");
                  scanf(``%f",&x0);
                  puts("\n enter upper limit of integral i.e. xn ");
                  scanf(``%f",&xn);
                  h=(xn-x0)/n;
                  s=y(x0)+y(xn);
                  for(i=1;i<=n-1;i++)</pre>
                     if(i%3!=0)
                       s1+=3*y(x0+i*h);
                     else
                        s2+= 2*y(x0+i*h);
                  s=s+s1+s2;
                  sum=s*(3*h/8);
                  printf("\n Value of Integral is %.3lf\n",sum);
                  qetch();
           Output: SIMPSON 3/8th RULE
           enter number of subdivisions i.e. n
           6
           enter lower limit of integral i.e. x0
           0
           enter upper limit of integral i.e. xn
           1
           Value of Integral is 0.693
```



.C.

#### LAB ASSIGNMENT : SIMPSON'S 3/8TH RULE

**1.** Write a C program to solve the integral  $\int_0^1 \frac{dx}{1+x^2}$  using Simpson 3/8 rule.

**Hint :** Define function y = 1/(1 + x \* x) **Input :**  $x_0 = 0, x_1 = 1, n = 6$ **Output :** Value of integral = 0.785396

**2.** Write a C program to solve the integral  $\int_0^6 (1+x^2) dx$  using Simpson 3/8th rule.

**Hint :** Define function y (x) = (1 + x \* x) **Input :**  $x_0 = 0$ ,  $x_n = 6$ , n = 6**Output :** Value of integral = 78

#### 4. BOOLE'S RULE

#### SYMBOLS USED

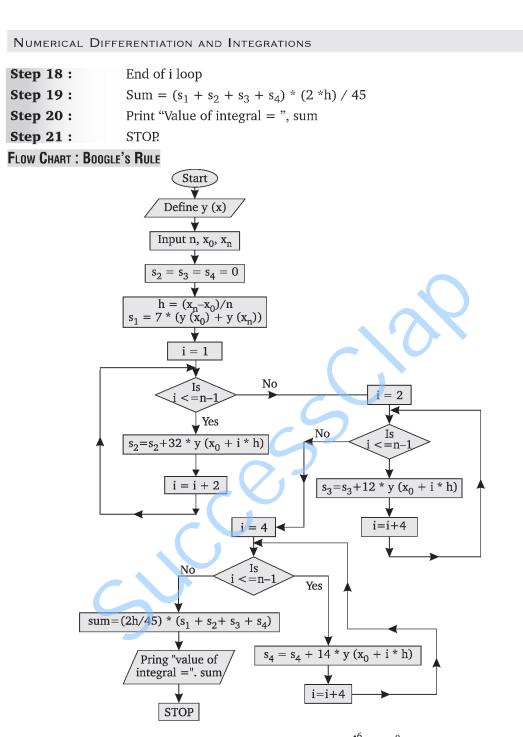
520

- n = number of subdivision
- $x_0 =$ lower limit of integral
- $x_n = upper limit of integral$
- h = length of subinterval
- y(x) = function to be integrated

# ALGORITHM : BOOLE'S RULE

Start
Define function y (x)
Input n , x <sub>0</sub> , x <sub>n</sub>
$s_2 = s_3 = s_4 = 0$
$h = (x_n - x_0)/n$
$s_1 = 7 * y (x_0) + y (x_n)$
For $i = 1$ to $(n - 1)$
$s_2 = s_2 + 32 * y (x_0 + i * h)$
i = i + 2
End of i loop
For $i = 2$ to $(n - 1)$
$s_3 = s_3 + 12 * y (x_0 + i * / h)$
i = i + 4
End of i loop
For $o = 4$ to $(n - 1)$
$s_4 = s_4 + 14 * y (x_0 + i * h)$
i = i + 4

521



PROGRAM: Following is a C program to solve the integral \$\int\_0^6 (1+x^2) dx\$ by the Boole's rule.
 // BOOLE's RULE
 #include<stdio.h>
 #include<conio.h>

```
NUMERICAL METHODS IN SCIENCE AND ENGINEERING
522
          float y(float x)
          {
             return (1+x^*x);
           }
           void main()
          {
                float x0,xn,h,s1,s2=0,s3=0,s4=0,s,sum;
                 int i,n;
                 clrscr();
                 puts("\n enter number of subdivisions i.e. n ");
                 scanf("%d", &n);
                 puts("\n enter lower limit of integral i.e. x0 ");
                 scanf("%f",&x0);
                puts("\n enter upper limit of integral i.e. xn ");
                 scanf("%f",&xn);
                 h=(xn-x0)/n;
                 s1=7*y(x0)+y(xn);
                 for(i=1;i<=n-1;i+=2)
                       s2+=32*y(x0+i*h);
                 for(i=1;i<=n-1;i+=4)
                      s3+= 12*y(x0+i*h);
                 for(i=4;i<=n-1;i+=4)</pre>
                       s4+=14*y(x0+i*h);
                 s=s1+s2+s3+s4;
                 sum=s*(2*h/45);
                 printf("\n Value of Integral is %.3lf\n",sum);
                 getch();
          Output: BOOLE's RULE
```

```
enter number of subdivisions i.e. n
8
enter lower limit of integral i.e. x0
0
enter upper limit of integral i.e. xn
6
Value of Integral is 67.450
```

NUMERICAL DIFFERENTIATION AND INTEGRATIONS

#### LAB ASSIGNMENT : BOOLE'S RULE

1. Write a C program to solve the integral  $\int_0^4 \frac{dx}{1+x^2}$  by Boole's rule. Divide the interval into 4 equal parts.

C'a

Hint : Define a function y (x) = 1/(1 + x \* x)Input :  $x_0 = 0, x_n = 4, n = 4$ Output : Value of integral = 1.28941

2. Write a C program to solve the integral  $\int_{1}^{7} \frac{1}{x} dx$  by Boole's rule. Hint : Define a function y (x) = (1/x) Input : x<sub>0</sub> = 1, x<sub>n</sub> = 7, n = 8 Output : Value of integral = 1.949018

#### 5. WEDDLE'S RULE

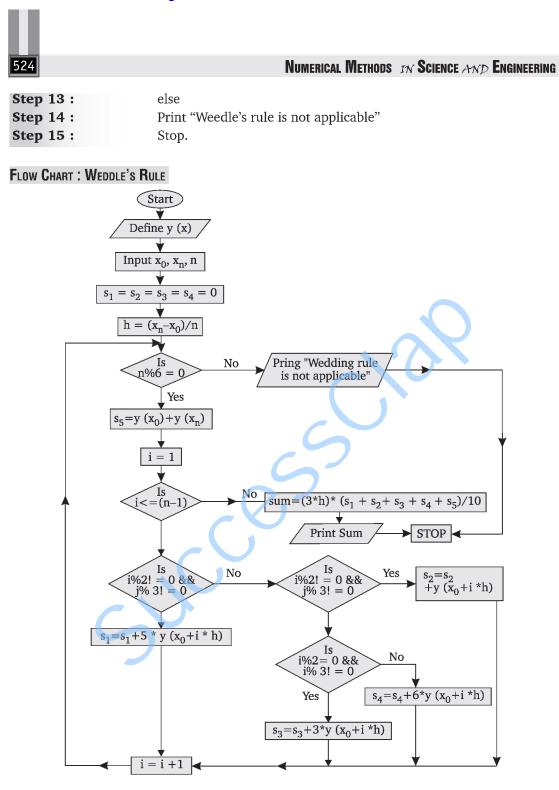
#### Symbols Used

- n = number of subdivision
- $x_0 = lower limit of integral$
- $x_n = upper limit of integral$
- h = length of subinterval
- y(x) = function to be integrated

### **ALGORITHM : WEDDING'S RULE**

Start
Define function y (x)
Input x <sub>0</sub> , x <sub>n</sub> , n
$s_1 = s_2 = s_3 = s_4 = 0$
$h = (x_n - x_0)/n$
If $(n \% 6 = 0)$
$s_5 = y (x0) + y (xn)$
For $i = 1$ to $n - 1$
If $(1\% 2! = 0 \text{ and } i\% 3! = 0)$
$s_1 = s_1 + 5 * y (x_0 + i * h)$
else
if $(i\%2 = 0 \text{ and } i\%3! = 0)$
$s_2 = s_2 + y (x_0 + i * h)$
else
if $(i\%2 = 0 \text{ and } i\%3 = 0)$
$s_3 = s_3 + 3 * y (x_0 + i * h)$
else
$s_4 = s_4 + 6 * y (x_0 + i * h)$
End of i loop
Sum = $3 * h * (s_1 + s_2 + s_3 + s_4 + s_5)/10$
print "Value of integral = ", sum





```
525
 NUMERICAL DIFFERENTIATION AND INTEGRATIONS
           Following is a C program to solve the integral \int_0^6 (1+x^2) dx by the Weddle's rule.
PROGRAM:
            // WEDDLE's RULE
           #include<stdio.h>
           #include<conio.h>
           float y(float x)
           {
              return (1+x*x);
            }
            void main()
           {
                 float x0, xn, h, s1=0, s2=0, s3=0, s4=0, s5, s, sum;
                 int i,n;
                 clrscr();
                 puts("\n enter number of subdivisions i.e. n ");
                  scanf("%d",&n);
                 puts("\n enter lower limit of integral i.e. x0 ");
                 scanf("%f",&x0);
                 puts("\n enter upper limit of integral i.e. xn ");
                  scanf("%f",&xn);
                 h=(xn-x0)/n;
                 if(n%6==0)
                    s5=y(x0)+y(xn);
                  else
                    {printf("\n Weddle's rule is not applicable\n");
                     goto end;
                    }
                  for (i=1; i<=n-1; i++)
                       if (i%2!=0 && i%3!=0)
                          s1+=5*y(x0+i*h);
                       else
                           if(i%2==0 && i%3=0)
                             s2+=y(x0+i*h);
                           else
                               if(i%2==0 && i%3==0)
                                     s3 += 3*y(x0+i*h);
                                else
                                       s4+=6*y(x0+i*h);
                    }
                 s=s1+s2+s3+s4+s5;
                 sum=s*(3*h/10);
                  printf("\n Value of Integral is %.3lf\n",sum);
```

```
NUMERICAL METHODS IN SCIENCE AND ENGINEERING
526
            end:
                    getch();
            }
            Output: WEDDLE's RULE
             enter number of subdivisions i.e. n
            6
             enter lower limit of integral i.e. x0
            0
             enter upper limit of integral i.e. xn
            6
             Value of Integral is 78.000
LAB ASSIGNMENT : WEDDLE'S RULE
   1. Write a C program to solve the integral \int_0^1 \frac{dx}{1+x^2} by Weddle's rule. Divide the range
      into six equal parts.
      Hint : Define function y(x) = 1/(1 + x * x)
          Input : x_0 = 0, x_n = 1, n = 6
          Output : Value of integral = 0.7854
  2. Write a C program to solve the integral \int_0^{1.5} \frac{x^3}{e^x - 1} dx by dividing the interval into six
```

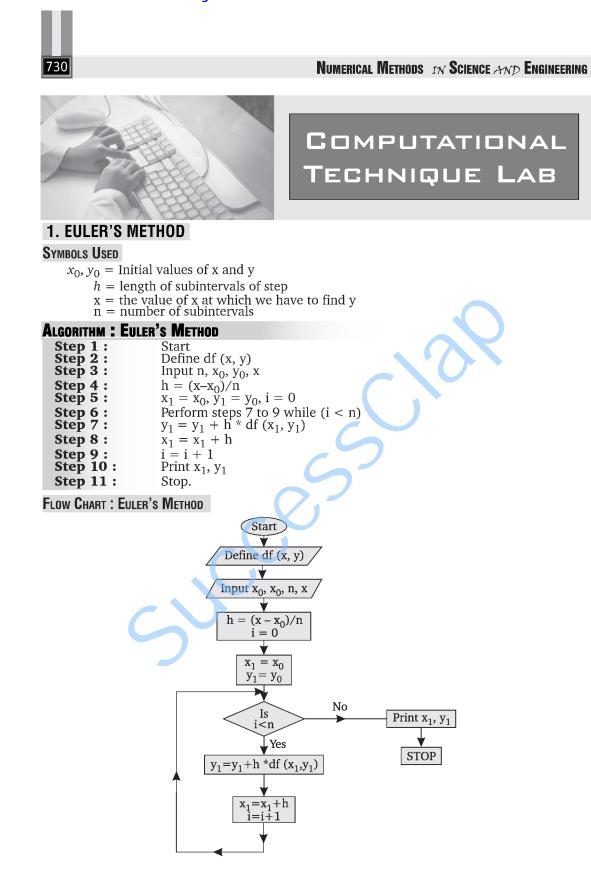
equal parts using Weddle's rule. **Hint** : Define function y(x) = (x \* x \* x)/exp(x) - 1

**Input :**  $x_0 = 0, x_n = 1.5, n = 6$ **Output :** Value of integral = 0.6155

**3.** Write a C program to solve the integral  $\int_0^5 \frac{dx}{4x+5}$  by Weddle's rule.

**Hint** : Define function y(x) = 1/(4 \* x + 5)**Input** :  $x_0 = 0, x_0 = 5, n = 12$ **Output :** Value of integral = 0.4023.

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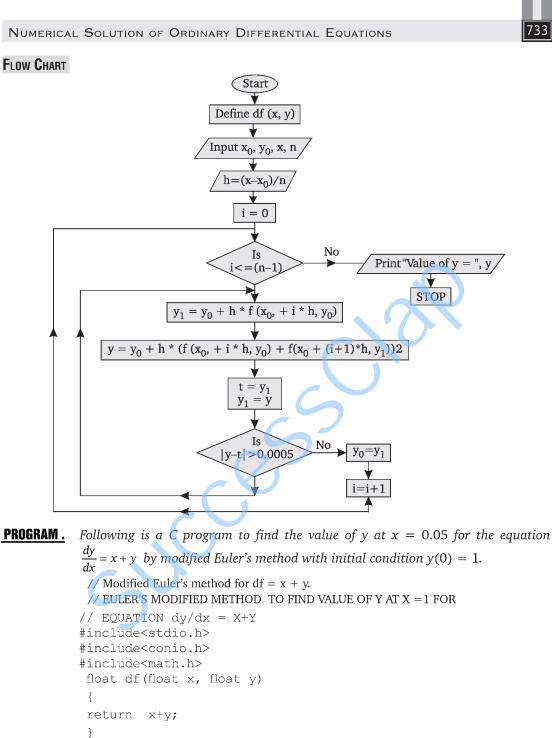


```
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NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS
PROGRAM: Following is a C program to find the value of y at x for the equation
           \frac{dy}{dx} = df(x, y) = x + y by the Euler's method.
            // EULER'S METHOD TO FIND VALUE OF Y AT X=1 FOR EQUATION dy/dx = X+Y
          #include<stdio.h>
          #include<conio.h>
           float df(float x, float y)
            {
            return x+y;
            }
           void main()
              {
              clrscr();
              float x0,y0,h,x,x1,y1,n,i;
              puts("\n enter value of x0 ");
              scanf("%f", &x0);
              puts("\n enter the value of y0 ")
              scanf("%f", &y0);
              puts ("\n enter the value of n");
              scanf("%f", &n);
              puts("\n enter the value of x ");
              scanf("%f", &x);
              h=(x-x0)/n;
              x1=x0;
              y1=y0;
             for(i=0;i<n;i++)</pre>
                y1+=h*df(x1,y1);
                x1+=h;
                printf(" When x =\$3.1f y = \$4.2f \n'', x1, y1);
                getch();
          Output: EULER'S METHOD TO FIND THE VALUE OF Y(X)=1 FOR
          EQUATION X+Y = (dy/dx)
          enter value of x0
          0
          enter value of y0
          1
```

732	Numerical Methods in Science and Engineering
	enter value of n
	10
	enter value of x
	1
	When $x = 1$ $y = 3.18748$
	gnment : Euler's Method
1.	Write a C program to find y for x = 0.1 for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initia
	condition $y = 1$ at $x = 0$ , dividing the interval into 5 equal parts.
	Hint : Define df = $(y-x)/(y+x)$ Input : n = 5, $x_0 = 0$ , $y_0 = 1$ , $x = 0.10$ Output : y (0.10) = 1.09283
2.	Write a C program for the Euler's method to solve the differential equation $\frac{dy}{dx} = y^2 - x^2$ for y (0.5) with y = 1 when x = 0.
	<b>Hint :</b> Define df = $y^* y - x^* x$
	<b>Input :</b> $x_0 = 0, y_0 = 1, x = 0.5, n = 5$
	<b>Output :</b> y (0.5) = 1.76393
	IFIED EULER'S METHOD
Symbols x	<b>USED</b> $_0, y_0 =$ Initial values of x and y
	= value of x at which we have to find y
	= number of subintervals
h	= step size

# ALGORITHM: MODIFIED EULER'S METHOD

Step 1 :	Start
Step 2 :	define df (x, y)
Step 3 :	Input x <sub>0</sub> , y <sub>0</sub> , x, n
Step 4 :	$\mathbf{h} = (\mathbf{x} - \mathbf{x}_0)/\mathbf{n}$
Step 5 :	for $i = 0$ to $n - 1$
Step 6 :	$y_1 = y_0 + h * f (x_0 + i * h, y_0)$
Step 7 :	$y = y_0 + h * (f (x_0 + i * h, y_0) + (f (x_0 + (i + 1) * h, y_1))/2$
Step 8 :	$t = y_1$
Step 9 :	$y_1 = y$
Step 10 :	Repeat step 7 to 9 until ( $ y-t  <=0.0005$ )
Step 11 :	$y_0 = y_1$
Step 12 :	End of for loop
Step 13 :	Print y
Step 14 :	Stop.



```
}
void main()
  {
     clrscr();
  float x0,y0,h,x,x1,y1,y,t;
  int n,i;
```

```
NUMERICAL METHODS IN SCIENCE AND ENGINEERING
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             puts("\n enter value of x0 ");
             scanf("%f", &x0);
             puts("\n enter the value of y0 ");
             scanf("%f", &y0);
             puts("\n enter the value of n ");
             scanf("%f", &n);
             puts("\n enter the value of x ");
             scanf("%f", &x);
             h = (x - x0) / n;
              for(i=0;i<n;i++)</pre>
              £
                 y1=y0+h*df(x0+i*h,y0);
                 do
                { y=y0+h*(df(x0+i*h,y0)+df(x0+(i+1)*h,y1))/2;
                    t=y1;
                   yl=y;
                 } while (fabs(y-t)>0.0005);
                y0=y1;
              }
               printf("\n When x = 83.1f
                                               %4.2f\n",x,y);
               qetch();
             }
          Output: EULER'S MODIFIED METHOD TO FIND VALUE OF Y(X)=1 FOR
                  EQUATION X+Y = (dy/dx)
          enter value of x0
          0
          enter value of y0
          1
          enter value of n
          5
          enter value of x
          .05
          When x = .05 y = 1.116388
```

LAB ASSIGNMENT : MODIFIED EULER'S METHOD

**1.** Write a C program to find y (2.2) by Euler's modified method for  $\frac{dy}{dx} = -xy^2$  where y (2) = 1. **Hint :** Define df = -x \* y \* y

Numerical Solution of Ordinary Differential Equations

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**Input :**  $x_0 = 2$ ,  $y_0 = 1$ , x = 2.2, n = 2**Output :** y (2.2) = 0.7018

**2.** Write a C program to find y (0.2) for  $\frac{dy}{dx} = \log_{10}(x+y)$  with initial condition y = 1 for x = 0 by modified Euler's method.

Hint : Define df = log 10 (x + y) Input :  $x_0 = 0, y_0 = 1, x = 0.2, n = 1$ Output : y (0.2) = 1.0082

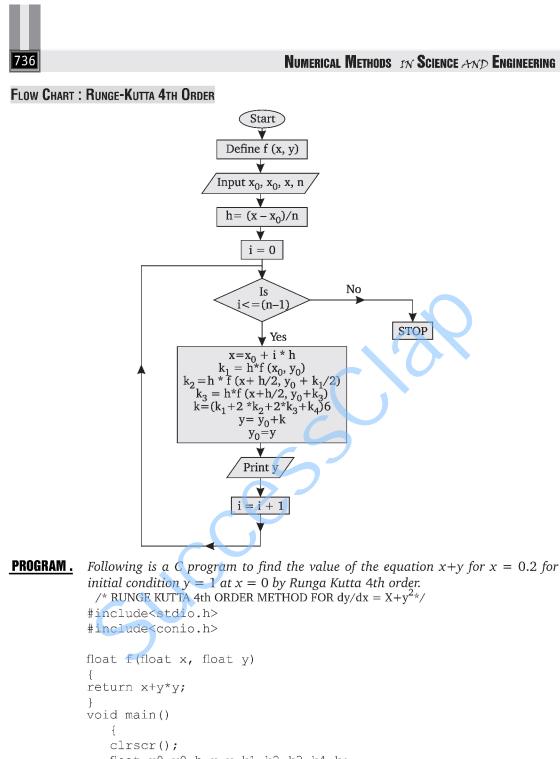
### 3. RUNGE-KUTTA METHOD

#### SYMBOLS USED

- $x_0$ ,  $y_0$  = initial values of x and y
- h = length of subinterval
- x = value of x at which we have to find y
- n = number of subdivision

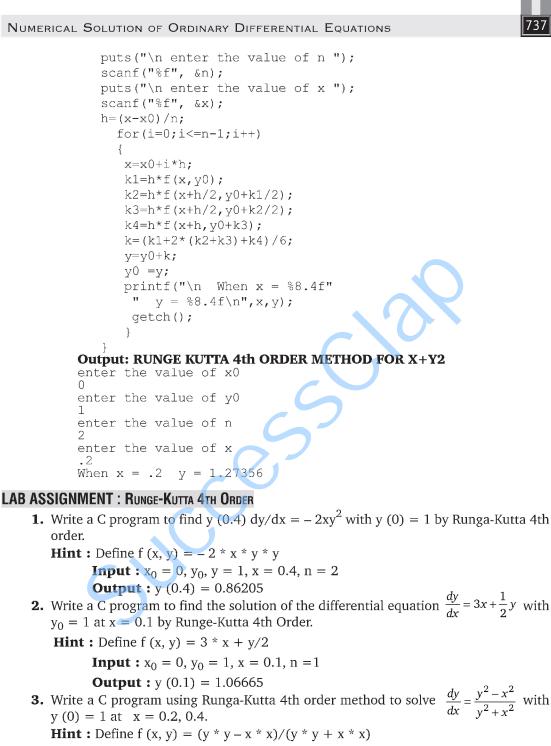
#### ALGORITHM : RUNGE-KUTTA 4TH ORDER

Step 1 :	Start
Step 2 :	Define f (x, y)
Step 3 :	Input n, x <sub>0</sub> , y <sub>0</sub> , x
Step 4 :	$h = (x - x_0)/n$
Step 5 :	For $i = 0$ to $(n - 1)$
Step 6 :	$\mathbf{x} = \mathbf{x}_0 + \mathbf{i} * \mathbf{h}$
Step 7 :	$k_1 = h * f (x, y_0)$
Step 8 :	$k_2 = h * f (x + h/2), y_0 + k_1/2)$
Step 9 :	$k_3 = h^* f (x + h/2, y_0 + k_2/2)$
<b>Step 10 :</b>	$k_4 = h * f (x + h, y_0 + k_3)$
Step 11 :	$\mathbf{k} = (\mathbf{k}_1 + 2 * \mathbf{k}_2 + 2 * \mathbf{k}_3 + \mathbf{k}_4)/6$
Step 12 :	$y + y_0 + k$
Step 13 :	$y_0 = y$
Step 14 :	Print y
Step 15 :	End of For Loop.
Step 16 :	Stop.



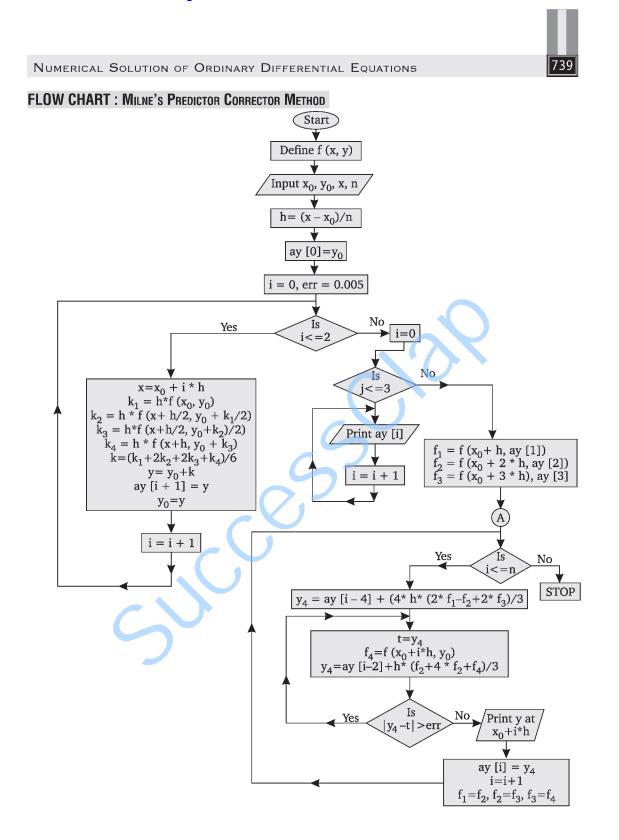
```
clrscr();
float x0,y0,h,x,y,k1,k2,k3,k4,k;
int i,n;
puts("\n enter the value of x0 ");
scanf("%f", &x0);
puts("\n enter the value of y0 ");
scanf("%f", &y0);
```

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- (a) **Input :**  $x_0 = 0$ ,  $y_0 = 1$ , x = 0.2, n = 1**Output :** y (0.2) = 1.1960
- (b) **Input :**  $x_0 = 0$ ,  $y_0 = 1$ , x = 0.4, n = 2**Output :** y(0.4) = 1.37527

```
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                                              NUMERICAL METHODS IN SCIENCE AND ENGINEERING
4. MILNE'S PREDICTOR CORRECTOR METHOD
SYMBOLS USED
    x_0, y_0 = initial values of x and y
    h = length of subinterval
    \mathbf{x} = the value of x at which we have to find y
    n = number of subdivision
    err = allowed error.
Algorithm : Milne's Predictor-Corrector Method
  Step 1:
               Start
  Step 2 :
              define f(x, y)
  Step 3 :
              Input x_0, y_0, x, n
  Step 4 :
              h(x - x_0)/n
  Step 5 :
              ay [0] = y_0, err = 0.0005
 Step 6 :
              For i to 2
  Step 7 :
             x = x_0 + i * h
             k_1 = h * f(x, y_0)
  Step 8 :
              k_2 = h * f (x + h/2, y_0 + k_1/2)
 Step 9 :
  Step 10 : k_3 = h * f (x + h/2, y_0 + k_2/2)
  Step 11 : k_4 = h * f (x + h, y_0 + k_3)
  Step 12 : k = (k_1 + 2 * k_2 + 2 * k_3 + k_4)/6
  Step 13 : y = y_0 + k
  Step 14 : ay [i + 1] = y
 Step 15 : y<sub>0</sub> = y
  Step 16 : End of For loop
  Step 17 : Print "Starting fair values for the Milne's method by Runga-Kutta method of
               order 4 are"
  Step 18 : For i = 0 to 3
  Step 19 : Print ay [i]
  Step 20 : f_1 = f(x_0 + h, ay [i])
  Step 21 : f_2 = f(x_0 + 2 * h, ay [2])
  Step 22 : f_3 = f(x_0 + 3 * h, ay [3])
  Step 23 : Repeat steps 24 to 31 until (i > n)
  Step 24 : y_4 = ay [i-4] + (4 * h * (2 * f_1 - f_2 + 2 * f_3))/3
  Step 25 : Repeat steps 26 to 28 until ([y_4 - t] < err)
  Step 26 : t = y<sub>4</sub>
  Step 27 : f_4 = f(x_0 + i * h, y_4)
  Step 28 : y_4 = ay [i-2] + h^*(f_2 + 4 * f_3 + f_4)/3
  Step 29 : print "Value of y at x =", x_0 + i * h, ay [i]
  Step 30 : i = i + 1
  Step 31 : f_1 = f_2, f_2 = f_3, f_3 = f_4
  Step 32 : Stop.
```



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```
NUMERICAL METHODS IN SCIENCE AND ENGINEERING
PROGRAM.
           Following program shows the Milne's Predictor-Corrector Method to find the
           approximate value of y for x = 0.4 for the equation dy/dx = xy + y^2 with initial
           condition y = 1 at x = 0 dividing the range into four equal parts.
          /* MILNE'S PREDICTOR CORRECTOR METHOD FOR dy/dx = X*Y+Y*Y */
           #include<stdio.h>
           #include<math.h>
           #include<conio.h>
           float f(float x,float y)
           return x*y+y*y;
           }
           void main()
           {
                clrscr();
                 float ay[5],x0,y0,x,y,h,t,k1,k2,k3,k4,k,err=.0005;
                 float f1, f2, f3, f4, y2, y4;
                  int i,n;
                  puts("Enter the value of x0,y0,x,n\n");
                  scanf(" %f %f %f %d", &x0,&y0,&x,&n);
                  h=(x-x0)/n;
                  ay[0]=y0;
             for(i=0;i<n;i++)</pre>
                  x=x0+i*h;
                  k1=h*f(x,y0);
                  k2=h*f(x+h/2,y0+k1/2);
                  k3=h*f(x+h/2,y0+k2/2);
                  k4=h*f(x+h,y0+k3);
                  k=(k1+2*(k2+k3)+k4)/6;
                  y=y0+k;
                  ay[i+1]=y;
                  у0=у;
              }
```

printf("\n Starting 4 values by runge -kutta mrthod are\n");

```
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NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS
          for(i=0;i<=3;i++)</pre>
            printf("\n y= %d= %.5f",i,ay[i]);
            f1=f(x0+h,ay[1]);
            f2=f(x0+2*h,ay[2]);
            f3=f(x0+3*h,ay[3]);
          while (i<=1)
             {
                 y_{4=ay[i-4]+(4*h*(2*f1-f2+2*f3))/3;}
                 do
                  {
                    t=y4;
                    f4=f(x0+i*h, y4);
                    y4=ay[i-2]+h*(f2+4*f3+f4)/3;
                    } while (fabs(y4-t)>err);
               printf("\n\n value of y at x=\%.2f=\%.5f", x0+i*h, y4);
                ay[i]=y4;
               i++;
                f1=f2; f2=f3, f3=f4;
              }
                getch();
                }
         Output: MILNE'S PREDICTOR CORRECTOR METHOD FOR X*Y+Y*Y
         Enter the value of x0, y0, x, n
```

```
0 1 .4 4
Starting 4 values by runge -kutta mrthod are
y= 0= 1.00000
y= 1= 1.11689
y= 2= 1.27739
y= 3= 1.50412
Value of y at x=0.40 is 1.83941
```



#### NUMERICAL METHODS IN SCIENCE AND ENGINEERING

#### LAB ASSIGNMENT : MILNE'S PREDICTOR CORRECTOR METHOD

1. Write a C program for Milne's Predictor Corrector method to find value of y for x = 0.5 for  $dy/dx = 2e^{x} - y$  with initial condition x = 0, y = 2 by dividing range into 5 equal parts.

**Hint :** Define f (x, y) =  $2 * \exp(x) - y$ **Input :**  $x_0 = 0, y_0 = 2, x = 0.5, n = 5$ 

**Output :** y (0.50) = 2.25525

**2.** Write a C program to find value of y at x = 0.5 of the differential equation dy/dx = x + y with initial condition y (0) = 1 by predictor-corrector method.

**Hint :** Define f(x, y) = x + y

**Input**:  $x_0 = 0$ ,  $y_0 = 1$ , x = 0.5, n = 5**Output**: y(0.5) = 1.7968

**3.** Write a C program for Milne's method to find y (1) for the equation  $dy/dx = x-y^2$  with initial condition y (0) = 0.

**Hint** : Define f(x, y) = x - y \* y

**Input** : x<sub>0</sub> = 0, y<sub>0</sub> = 0, x = 1, n = 5 **Output** : y (1) =0.45552

J.C.

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