UPSC CSE Mathematics: Previous Year Questions: Linear Algebra

2025

1) Can the set $\{(0,0,0,3),(1,1,0,0),(0,1,-1,0)\}$ be extended to form a basis of the vector space $\mathbb{R}^4$? Justify your answer.

2) Find the range, rank, kernel and nullity of the linear transformation $\mathrm{T}: \mathbb{R}^4 \to \mathbb{R}^3$ given by $\mathrm{T}(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{w}) = (\mathrm{x}-\mathrm{w}, \mathrm{y}+\mathrm{z}, \mathrm{z}-\mathrm{w})$.

3) Let $\mathrm{T}: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $\mathrm{T}(1,1,-1) = (1,0)$, $\mathrm{T}(4,1,1) = (0,1)$ and $\mathrm{T}(1,-1,2) = (1,1)$. Find T .

4) Reduce the following matrix to echelon form: $A = \begin{bmatrix} 2 & -2 & 2 & 1 \\ -3 & 6 & 0 & -1 \\ 1 & -7 & 10 & 2 \end{bmatrix}$

5) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & -2 & 3 \end{bmatrix}$$

6) Let P_n denote the vector space of all polynomials of degree $\leq n$ over $\mathbb{R}$. Verify that

$$\dim\left(\frac{P_4}{P_2}\right) = \dim P_4 - \dim P_2$$

2024

1) Let H be a subspace of R^4 spanned by the vectors $v_1 = (1,-2,5,-3)$. $v_2 = (2,3,1,-4)$, $v_3 = (3,8,\ -3,-5)$ Then find a basis and dimension of H, and extend the basis of H to a basis of R^4.

2) Let $T: R^3 \to R^3$ be a linear operator and $B = (v_1, v_2, v_3)$ be a basis of R^3 over R. Suppose that $Tv_1 = (1,1,0)$, $Tv_2 = (1,0,-1)$, $Tv_3 = (2,1,-1)$. Find a basis for the range space and null space of T.

3) Consider a linear operator T on $\mathbf{R}^3$ over R defined by T(x,y,z) = $(2x, 4x-y, 2x+3y-z)$. Is T invertible? If yes, justify your answer and find $\boldsymbol{T}^{-1}$.

4) Let $V = M_{2\times 2}(R)$ denote a vector space over the field of real numbers. Find the matrix of the linear mapping $\phi: V \to V$ given by $\phi(v) = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} v$ with respect to standard basis of $M_{2\times 2}(\mathbb{R})$, and hence find the rank of φ. Is ϕ invertible? Justify your answer.

5) Let $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ be a 3×3 matrix. Find the eigenvalues and the corresponding eigenvectors of A. Hence find the eigenvalues and the corresponding eigenvectors of A^{-15}, where $A^{-15} = (A^{-1})^{15}$.

2023

1) Let $V_1 = (2,-1,3,2), V_2 = (-1,1,1,-3)$ and $V_3 = (1,1,9,-5)$ be three vectors of the space $\mathbb{R}^4$. Does $(3,-1,0,-1) \in \text{span}\{V_1, V_2, V_3\}$? Justify your answer.

2) Find the rank and nullity of the linear transformation:
$T: \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(x,y,z) = (x+z, x+y+2z, 2x+y+3z)$

3) If the matrix of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ relative to the basis (1,0,0), (0,1,0), (0,0,1) is $\begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ then find the matrix of T relative to the basis $\{(1,1,1),(0,1,1),(0,0,1)\}$.

4) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$
(i) Verify the Cayley-Hamilton theorem for the matrix A.
(ii) Show that $A^n = A^{n-2} + A^2 - I$ for $n \geqslant 3$, where I is the identity matrix of order 3. Hence, find A^{40}.

5) Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ by reducing to row-reduced echelon form.

2022

1) Prove that any set of $\boldsymbol{n}$ linearly independent vectors in a vector space V of dimension $\boldsymbol{n}$ constitutes a basis for V.

2) Let $\mathrm{T}: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation and $\mathrm{T}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathrm{T}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 8 \end{pmatrix}$. Find $\mathrm{T}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

3) Let the set $\mathrm{P} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \begin{matrix} x - y - z = 0 \text{ and} \\ 2x - y + z = 0 \end{matrix} \right\}$
be the collection of vectors of a vector space $\mathbb{R}^3(\mathbb{R})$. Then
(i) prove that P is a subspace of $\mathbb{R}^3$.
(ii) find a basis and dimension of P.

4) Find a linear map $T: \mathbb{R}^2 \to \mathbb{R}^2$ which rotates each vector of $\mathbb{R}^2$ by an angle θ. Also, prove that for $\theta = \frac{\pi}{2}$, T has no eigenvalue in $\mathbb{R}$.

5) Find all solutions to the following system of equations by row-reduced method:

$$x_1 + 2x_2 - x_3 = 2 \qquad 2x_1 + 3x_2 + 5x_3 = 5 \qquad -x_1 - 3x_2 + 8x_3 = -1$$

2021

1) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then show that $A^2 = A^{-1}$ (without finding A^{-1}).

2) Find the matrix associated with the linear operator on $V_3(R)$ defined by $T(a, b, c) = (a + b, a - b, 2c)$ with respect to ordered basis $B = \{(0,1,1), (1,0,1), (1,1,0)\}$.

3) Show that $S = \{(x, 2y, 3x): x, y$ are real numbers $\}$ is a subspace of $R^3(R)$.
Find two bases of S. Also find the dimension of S.

4) Prove that the eigen vectors, corresponding to two distinct eigen values of a real symmetric matrix, are orthogonal.

5) For two square matrices A and B of order 2, show that trace $(AB) = \text{trace } (BA)$. Hence show that $AB - BA \neq I_2$, where I_2 is an identity matrix of order 2

6) Reduce the matrix to row-reduced echelon form and also find rank: $A = \begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$

7) Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, over the complex-number field.

2020

1) Consider the set V of all $n \times n$ real magic squares. Show that V is a vector space over R.Give examples of two distinct 2×2 magic squares.

2) Let $T: M_2(R)$ be the vector space of all 2×2 real matrices. Let $B = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}$.Suppose $T: M_2(R) \to M_2(R)$ is a linear transformation defined by $T(A) = BA$. Find the rank and nullity of T. Find a matrix A which maps to the null matrix.

3) Define an $n \times n$ matrix as $A = I - 2u.u^T$, where u is a unit column vector
   (i) Examine if A is symmetric.
   (ii) Examine if A is orthogonal.
   (iii) Show that trace $(A) = n - 2$.
   (iv) Find $A_{3\times3}$ when $u = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$

4) Let F be a subfield of complex numbers and T a function from $F^3 \to F^3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + 3x_3,\ 2x_1 - x_2,\ -3x_1 + x_2 - x_3)$. What are the conditions on a, b, c such that (a, b, c) be in the null space of T ? find the nullity of T.

5) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$
   a) Find AB b) Find $\det(A)$ and $\det(B)$
   b) Solve the linear equations: $x + 2z = 3 \quad 2x - y + 3z = 3 \quad 4x + y + 8z = 14$

2019

1) Let $T: R^2 \to R^2$ be a linear map such that $T(2,1) = (5,7)$ and $T(1,2) = (3,3)$ If A is the matrix corresponding to T with respect to the standard bases e_1, e_2, then find rank

2) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$ then show that $AB = 6I_3$. Use this result to solve the system of equations. $2x + y + z = 5,\ x - y = 0,\ 2x + y - z = 1$

3) Let A and B be two orthogonal matrices of same order and $\det A + \det B = 0$ Show that $A + B$ is a singular matrix.

4) Let $A = \begin{pmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{pmatrix}$

   a) Find the rank of matrix A
   b) Find the dimension of the subspace

$$V = \left\{ (x_1, x_2, x_3, x_4) \in R^4 \mid A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}$$

5) State the Cayley-Hamilton theorem. Use this theorem of find A^{100} where $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

2018

1) Let A be a 3×2 matrix and B a 2×3 matrix. Show that $C = A.B$ is a singular matrix.

2) Express basis vectors $e_1 = (1,0)$ and $e_2 = (0,1)$ as linear combination of $\alpha_1 = (2,-1)$ and $\alpha_2 = (1,3)$.

3) Show that if A and B are similar $n \times n$ matrices, then they have the same Eigen values.

4) For the linear equations $x + 3y - 2z = -1,\ 5y + 3z = -8, x - 2y - 5z = 7$ determine which of the following statements are true and which are false:

   a. The system has no solution.
   b. The system has a unique solution.
   c. The system has infinitely many solutions.

2017

1) Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is diagonal matrix.

2) Show that similar matrices have the same characteristic polynomial.

3) Suppose U and W are district four dimensional subspaces of a vector space V, where $\dim\ V = 6$. Find the possible dimensions of subspace $U \cap W$

4) Consider the matrix mapping $A: R^4 \to R^3$, where $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{pmatrix}$. Find a basis and dimension of the image of A and those of the kernel A.

5) Prove that distinct non-zero eigenvectors of a matrix are linearly independent.

6) Consider the following system of equation in x, y, z

$$x + 2y + 2z = 1, \qquad x + ay + 3z = 3, \qquad x + 11y + az = b$$

a. For which values of a does the system have a unique solution?
b. For which of values (a, b) does the system have more than one solution?

2016

1) Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

2) If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ then find $A^{14} + 3A - 2I$.

3) Using elementary row operation find the condition that the linear equations have a solution

$$x - 2y + z = a$$
$$2x + 7y - 3z = b$$
$$3x + 5y - 2z = c$$

4) If $W_1 = \{(x, y, z) \mid x + y - z = 0\}$, $W_2 = \{(x, y, z) \mid 3x + y - 2z = 0\}$, $W_3 = \{(x, y, z) \mid x - 7y + 3z = 0\}$ then find $\dim(W_1 \cap W_2 \cap W_3)$ and $\dim(W_1 + W_2)$

5) If $M_2(R)$ is space of real matrices of order 2×2 and $P_2(x)$ is the space of real polynomials of degree at most 2 , then find the matrix representation of $T: M_2(R) \to P_2(x)$ such that $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + c + (a - d)x + (b + c)x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$, Further find null space of T

6) If $T: P_2(x) \to P_3(x)$ is such that $T(f(x)) = f(x) + 5\int_0^x f(t)dt$, then choosing $\{1,\ 1 + x,\ 1 - x^2\}$ and $\{1,\ x,\ x^2,\ x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively find the matrix of T.

7) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the eigenvalues and Eigenvectors of A.

8) Prove that the eigenvalues of a Hermitian matrix are all real.

9) If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation $T: P_2(x) \to P_2(x)$ with respect to bases $\{1 - x,\ x(1 - x),\ x(1 + x)\}$ and $\{1,\ 1 + x,\ 1 + x^2\}$, then find T.

2015

1) The vectors $V_1 = (1,1,2,4), V_2 = (2,-1,-5,2), V_3 = (1,-1,-4,0)$ and $V_4 = (2,1,1,6)$ are linearly independent. Is it true? Justify your answer.

2) Reduce the matrix to row echelon form and find its rank: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$

3) If matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then find A^{30}

4) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

5) Let $V = R^3$ and $T \in A(V)$, for all $a_i \in A(V)$, be defined by

$T(a_1, a_2, a_3) = (2a_1 + 5a_2 + a_3, \ -3a_1 + a_2 - a_3, \ a_1 + 2a_2 + 3a_3)$. What is the matrix T relative to the basis $V_1 = (1,0,1), V_2 = (-1,2,1), V_3 = (3,-1,1)$?

6) Find the dimension of the subspace of R^4, spanned by the set $\{(1,0,0,0), (0,1,0,0), (1,2,0,1), (0,0,0,1)\}$. Hence find its basis.

2014

1) Find one vector in R^3 which generates the intersection of V and W, where V is the xy-plane and W is the space generated by the vectors $(1,2,3)$ and $(1,-1,1)$

2) Using elementary row or column operations, find rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

3) Let V and W be the following subspaces of R^4

$V = \{(a,b,c,d): \ b - 2c + d = 0\}$ and $W = \{(a,b,c,d): a = d, b = 2c\}$. Find a basis and the dimension of (i) V (ii) W (iii) $V \cap W$

4) Investigate the values of λ and μ so that the equations

$x + y + z = 6, \ \ x + 2y + 3z = 10, \ \ x + 2y + \lambda z = \mu$ have (i) no solution (ii) unique solution, (iii) an infinite number of solutions.

5) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse. Also, find the matrix represented by $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

6) Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the Eigen values of A and the corresponding Eigen vectors.

7) Prove that Eigen values of a unitary matrix have absolute value 1.

2013

1) Find the inverse of the matrix: $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$ by using elementary row operations. Hence solve the linear equations $x + 3y + z = 10$, $2x - y + 7z = 12,\ 3x + 2y - z = 4$

2) Let A be a square matrix and $A*$ be its adjoint, show that the Eigen values of matrices AA^* and $A*A$ are real. Further show that $\text{trace}\ (AA^*) = \text{trace}\ (A^*A)$

3) Let P_n denote the vector space of all real polynomials of degree at most n and $T: P_2 \to P_3$ be linear transformation given by $T(f(x)) = \int_0^x p(t)dt, p(x) \in P_2$. Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1 + x^2, 1 + x^3\}$ of P_2 and P_3 respectively. Also find the null space of T

4) Let V be an n-dimensional vector space and $T: V \to V$ be an invertible linear operator. If $\beta = \{X_1, X_2, \dots X_n\}$ is a basis of V, show that $\beta' = \{TX_1, TX_2, \dots TX_n\}$ is also a basis of V

5) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where $\omega(\neq 1)$ is a cube root of unity. If $\lambda_1, \lambda_2, \lambda_3$ denote the Eigen values of A^2, show that $|\lambda|_1 + |\lambda_2| + |\lambda_3| \leq 9$

6) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 5 & 8 & 12 & 17 \\ 3 & 5 & 8 & 17 & 23 \\ 8 & 12 & 17 & 23 & 30 \end{bmatrix}$

7) Let A be a Hermitian matrix having all distinct Eigen values $\lambda_1, \lambda_2, \dots . \lambda_n$. If $X_1, X_2, \dots X_n$ are corresponding Eigen vectors then show that the $n \times n$ matrix C whose k^{th} column consists of the vector X_n is nonsingular.

8) Show that the vectors $X_1 = (1, 1 + i, i), X_2 = (i, -i, 1 - i)$ and $X_3 = (0, 1 - 2i, 2 - i)$ in C^3 are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers.

2012

1) Prove or disprove the following statement:

If $B = \{b_1, b_2, b_3, b_4, b_5\}$ is a basis for $\mathbb{R}^5$ and V is a two-dimensional subspace of $\mathbb{R}^5$, then V has a basis made of two members of B.

2) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by
$T(\alpha, \beta, \gamma) = (\alpha + 2\beta - 3\gamma,\ \ 2\alpha + 5\beta - 4\gamma,\ \ \alpha + 4\beta + \gamma)$. Find a basis and the dimension of the image of T and the kernel of T

3) Let V be the vector space of all 2×2 matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V ? Justify your answer?

4) Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation: $x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$

$$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0 \qquad 3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$$

5) Consider the linear mapping $f: \mathbb{R}^2 \to \mathbb{R}^2$ by $f(x,y) = (3x+4y, 2x-5y)$. Find the matrix A relative to the basis $(1,0), (0,1)$ and the matrix B relative to the basis $(1,2), (2,3)$

6) If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of $\text{Adj } A$

7) Let $H = \begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $D = P^T H \bar{P}$ is diagonal.

2011

1) Let A be a non-singular $n \times n$, square matrix. Show that $A.(\text{adj } A) = |A|.I_n$ Hence show that $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

2) Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$

Solve the system of equations given by $AX = B$. Using the above, also solve the system of equations $A^T X = B$ where A^T denotes the transpose of matrix A.

3) Let $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ be the Eigen values of a $n \times n$ square matrix A with corresponding Eigen vectors $X_1, X_2, \ldots\ldots X_n$. If B is a matrix similar to A , show that the Eigen values of B is same as that of A. Also find the relation between the Eigen vectors of B and Eigen vectors of A.

4) Show that the subspaces of $\mathbb{R}^3$ spanned by two sets of vectors $\{(1,1,-1), (1,0,1)\}$ and $\{(1,2,-3), (5,2,1)\}$ are identical. Also find the dimension of this subspace.

5) Find the nullity and a basis of the null space of the linear transformation $A: \mathbb{R}^{(4)} \to \mathbb{R}^{(4)}$ given by the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

6) Show that the vectors $(1,1,1), (2,1,2)$ and $(1,2,3)$ are linearly independent in $\mathbb{R}^{(3)}$. Let $\mathbb{R}^{(3)} \to \mathbb{R}^{(3)}$ be a linear transformation defined by $T(x,y,z) = (x+2y+3z, x+2y+5z, 2x+4y+6z)$ Show that images of above vectors are linearly dependent. Given reason for the same.

7) Let $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3×3. Find the Eigen values of the matrix B^3 where $B = C^{-1}AC$.

8) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}$

Using this, show that A is non-singular and find A^{-1}.

2010

1) If $\lambda_1, \lambda_2, \dots . \lambda_3$ are the Eigen values of matrix A $= \begin{bmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 44 & 2 & 28 \end{bmatrix}$ show $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \leqq \sqrt{1949}$

2) What is the null space of the differentiation transformation $\frac{d}{dx}: P_n \to P_n$ where P_n is the space of all polynomials of degree $\leq n$ over the real numbers? What is the null space of the second derivative as a transformation of P_n? What is the null space of the kth derivative P_n ?

3) Let. $M = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$. Find the unique linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ so that M is the matrix of T with respect to the basis $\beta = \{v_1 = (1,0,0) \ v_2 = (1,1,0) \ v_3 = (1,1,1)\}$ of $\mathbb{R}^3$ and $\beta' = \{w_1 = (1,0), w_2 = (1,1)\}$ of $\mathbb{R}^2$. Also find $T(x, y, z)$.

4) Let A and B be $n \times n$ matrices over reals. Show that I -BA is invertible if $I - AB$ is invertible. Deduce that AB and AB have the same Eigen values.

5) In the n-space R^n , determine whether or not the $\{e_1 - e_2, e_2 - e_3, \dots \dots, e_{n-1} - e_n, e_n - e_1\}$ is linearly independent.

6) Let T be a linear transformation from a vector V space over reals into V such that $T - T^2 = I$. Show that T is invertible.

2009

1) Find a Hermitian and Skew Hermitian matrix each whose sum is the matrix

$$\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2+3i & 2 \\ -i+1 & 4 & 5i \end{bmatrix}.$$

2) Prove that the set V of the vectors (x_1, x_2, x_3, x_4) in $\mathbb{R}^4$which satisfy the equation $x_1 + x_2 + 2x_3 + x_4 = 0$ and $2x_1 + 3x_2 - x_3 + x_4 = 0$, is a subspace of $\mathbb{R}^4$. What is dimension of this subspace? Find one of its bases.

3) Let $\beta = \{(1,1,0)(1,0,1)(0,1,1)\}$ and $\beta' = \{(2,1,1), (1,2,1)(-1,1,1)\}$ be the two ordered bases of R^3. Then find a matrix representing the linear transformation $T: R^3 \to R^3$ which transforms β into β'. Use this matrix representation to find $T(x)$, where $x = (2,3,1)$.

4) Find a 2×2 real matrix A which is both orthogonal and skew-symmetric. Can there exist a 3×3 real matrix which is both orthogonal and skew-symmetric? Justify your answer.

5) Let $L: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by $L(x_1, x_2, x_3, x_4)) = (x_3 + x_4 - x_1 - x_2, \ x_3 - x_2, \ x_4 - x_1)$. Then find the rank and nullity of L. Also, determine null space and range space of L.

6) Prove that the set V of all 3×3 real symmetric matrices form a linear subspace of the space of all 3×3 real matrices. What is the dimension of this subspace? Find at least one of the bases for V.

2008

1) Show that the matrix A is invertible if and only if the $\text{adj}\ (A)$ is invertible. Hence find $|\text{adj}\ (A)|$

2) Let S be a non-empty set and let V denote the set of all functions from S into R. Show that V is vector space with respect to the vector addition $(f+g)(x) = f(x) + g(x)$ and scalar multiplication $(c.f)(x) = cf(x)$

3) Show that $B = \{(1,0,0), (1,1,0), (1,1,1)\}$ is a basis of. R^3. Let $T: R^3 \to R^3$ be a linear transformation such that $T(1,0,0) = (1,0,0), T(1,1,0) = (1,1,1)$ and $T(1,1,1) = (1,1,0)$.Find $T(x,y,z)$

4) Let A be a non-singular matrix. Show that if $I + A + A^2 + \cdots .. + A^n = 0$ then $A^{-1} = A^n$.

5) Find the dimension of the subspace of R^4 spanned by the set $\{(1,0,0,0)(0,1,0,0)(1,2,0,1), (0,0,0,1)\}$. Hence find a basis for the subspace.

2007

1) Let S be the vector space of all polynomials, $p(x)$ with real coefficients, of degree less than or equal to two considered over the real field R, such that $p(0)$=0 and $p(1) = 0$.Determine a basis for S and hence its dimension

2) Let T be the linear transformation from R^3 to R^4define by
$T(x_1, x_2, x_3) = (2x_1 + x_2 + x_3,\ \ x_1 + x_2,\ \ x_1 + x_3,\ \ 3x_1 + x_2 + 2x_3)$ for each $(x_1, x_2, x_3) \in R^3$ Determine a basis for Null space of T. What is the dimension of Range space of T?

3) Let W be the set of all 3×3 symmetric matrices over R. Does it from a subspace of the vector space of the 3×3 matrices over R ? In case it does, construct a basis for this space and determine its dimension

4) Consider the vector space $X := \{p(x)\}$, $p(x)$is a polynomial of degree less than or equal to 3 with real coefficients, over the real field R. Define the map $D: X \to X$ by
$(Dp)(x) := p_1 + 2p_2x + 3p_3x^2$ where $p(x) = p_0 + p_1x + p_2x^2 + p_3x^3$. Is D a linear transformation on X ? If it is, then construct the matrix representation for D with respect to the order basis $\{1, x, x^2, x^3\}$ for X.

5) Reduce the quadratic form $q(x,y,z) := x^2 + 2y^2 - 4xz + 4yz + 7z^2$ to canonical form. Is q positive definite?

2006

1) Let V be the vector space of all 2×2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V.

2) State Cayley-Hamilton theorem and using it, find the inverse of $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

3) If $T: R^2 \to R^2$ is defined by $T(x,y) = (2x - 3y, x + y)$. Compute the matrix of T relative to the basis$\beta\ = \{(1,2), (2,3)\}$

4) Using elementary row operations, find the rank of the matrix $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$.

5) Investigate for what values of and the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

Have-

a. no solution;
b. a unique solution;
c. infinitely many solutions

6) Find the quadratic form $q(x, y)$ corresponding to the symmetric matrix $A = \begin{pmatrix} 5 & -3 \\ -3 & 8 \end{pmatrix}$

Is this quadratic form positive definite? Justify your answer.

2005

1) Find the values of k for which the vectors $(1,1,1,1), (1,3,-2,k),\ (2,2k-2,-k-2,3k-1)$ and $(3,k+2,-3,2k+1)$ are linearly independent in R^4.

2) Let V be the vector space of polynomials in x of degree $\leq n$ over R. Prove that the set $\{1, x, x^2, \dots, x^n\}$ is a basis for V . Extend this basis so that it becomes a basis for the set of all polynomials in x.

3) Let T be a linear transformation on R^3 whose matrix relative to the standard basis of R^3 is $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ Find the matrix of T relative to the basis$\beta = \{(1,1,1), (1,1,0), (01,1)\}$.

4) Find the inverse of matrix using elementary row operations only: $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

5) If S is skew-Hermitian matrix, then show that show $A = (I + S)(I - S)^{-1}$ is a unitary matrix. Also show thatevery unitary matrix can be expressed in the above form provided -1 is not an Eigen value of A.

6) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ to the sum of squares. Also find the corresponding linear transformation, index and signature.

2004

1) Let S be space generated by the vectors $\{(0,2,6), (3,1,6), (4,-2,-2)\}$.What is the dimension of the space S ? Find a basis for S.

2) Show that $f: R^3 \to IR$ is la linear transformation, where $f(x, y, z) = 3x + y - z$. what is the dimension of the Kernel? Find a basis for the Kernel.

3) Show that the linear transformation form R^3 to R^4 which is represented by the matrix $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$ is one-to-one. Find a basis for its image.

4) Verify whether the following system of equation is consistent

$$x + 3z = 5$$
$$-2x + 5y - z = 0$$
$$-x + 4y + z = 4$$

5) Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$. Hence find A^{-1} and A^6

6) Define a positive definite quadratic form. Reduce the quadratic form $x_1^2 + x_3^2 + 2x_1x_2 + 2x_2x_3$ to canonical form. Is this quadratic form positive definite?

2003

1) Let S be any non-empty subset of a vector pace V over the field F. Show that the set $\{a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_{n:} : a_1, a_2, \ldots., a_n \in F, \alpha_1, \alpha_2, \ldots \ldots, \alpha_n \in S, n \in N\}$ is the subspace generated by S.

2) If $= \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then find matrix $2A^{10} - 10A^9 + 14A^8 - 6A^7 - 3A^6 + 15A^5 - 21A^4 + 9A^3 + A - 1.$

3) Prove that the Eigen vectors corresponding to distinct Eigen values of a square matrix are linearly independent.

4) If H is a Hermitian matrix, then show that $A = (H + iI)^{-1}(H - iI)$ is a unitary matrix. Also, so that every unitary matrix can be expressed in this form, provided 1 is not Eigen value of A.

5) If $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ then find a diagonal matrix D and a matrix B such that $A = BDB'$ where B' denotes the transpose of B.

6) Reduce the quadratic form given below to canonical form and find its rank and signature $x^2 + 4y^2 + 9z^2 + u^2 - 12yz + 6zx - 4xy - 2xu - 6zu.$

2002

1) Show that the mapping $T: R^3 \to R^3$ where $T(a, b, c) = (a - b, b - c, a + c)$ is linear and non-singular

2) A square matrix A is non-singular if and only if the constant term in its characteristic polynomial is different from zero.

3) Let $R^5 \to R^5$ be a linear mapping given by $T(a, b, c, d, e) = (b - d,\ \ d + e,\ \ b,\ 2d + e,\ b + e)$ Obtain bases for its null space and range space.

4) Let A be a real 3×3 symmetric matrix with Eigen values 0, 0 and 5. If the corresponding Eigen-vectors are $(2,0,1)$, $(2,1,1)$ and $(1,0,-2)$, then find the matrix A.

5) Solve the following system of linear equations

$$x_1 - 2x_2 - 3x_3 + 4x_4 = -1$$
$$-x_1 + 3x_2 + 5x_3 - 5x_4 - 2x_5 = 0$$
$$2x_1 + x_2 - 2x_3 + 3x_4 - 4x_5 = 17,$$

6) Use Cayley-Hamilton theorem to find the inverse of matrix: $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

2001

1) Show that vectors $(1,0-1),(0,-3,2)$ and $(1,2,1)$ form a basis for the vector space $R^3(R)$

2) If λ is characteristic root of non-singular matrix A then prove $\frac{|A|}{\lambda}$ is characteristic root of Adj.A

3) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ show that for every integer $n \geq 3, A^n = A^{n-2} + A^2 - I$. Hence find A^{50}.

4) When is a square matrix A said to be congruent to a square matrix B ? Prove that every matrix congruent to skew-symmetric matrix is skew symmetric.

5) Determine an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix, where
$$A = \begin{pmatrix} 7 & 4 & 4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{pmatrix}$$

6) Show that the real quadratic form $\phi = n(x_1^2 + x_2^2 + \cdots + x_n^2) - (x_1 + x_2 + \cdots + x_n)^2$ in n variables is positive semi-definite.