CSE 2023, CSE 2023, AIR 19, CSE 2023





Best Coaching for UPSC SYP MATHEMATICS

28th July 2023

Test Series 2023

Topic: 05 Ordinary Differential Equations

Instructions:

Time: 90 Minutes

Maximum Marks: 150

All questions are compulsory

Each question carries Equal marks

Assume suitable data if considered necessary



1) Find the orthogonal trajectories of the family of curves $x^2/(a^2 + \lambda) + y^2/(b^2 + \lambda) = 1$, where λ is a parameter.

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1 - 0$$

differtique w.r.t x

$$\Rightarrow \lambda = \frac{26^2 + 99^1 a^2}{-(x+99')}$$

put in equation ()

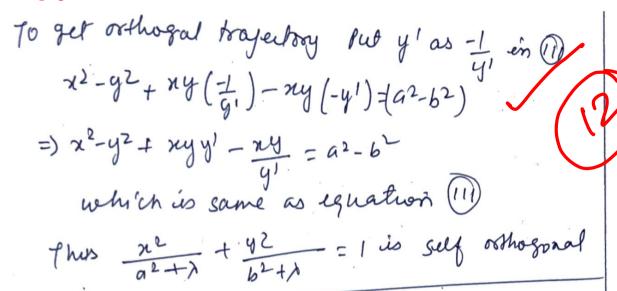
$$= \frac{x^{2}}{a^{2} - xb^{2} + yy^{1}a^{2}} + \frac{y^{2}}{b^{2} - xb^{2} + yy^{1}a^{2}} = 1$$

$$= \frac{x^{2}}{x + yy} + \frac{y^{2}}{x + yy}$$

=)
$$\frac{\chi^{2}(\chi_{1}, y, y_{1})}{\chi(a^{2}-b^{2})} + \frac{\chi^{2}(\chi_{1}, y, y_{1})}{(b^{2}-a^{2})\chi(y_{1})} = 1$$

=) ·
$$(x^2 + xyy) + - (xy + y^2y) = 92 - 62$$

$$=) \left[x^2 - y^2 + xyy' - \frac{xy}{y'} = a^2 - b^2 \right] - (11)$$



Solve the following differential equations:

$$p^3 - 4xyp + 8y^2 = 0$$

$$4xyp = p^3 + 8y^2$$

$$\Rightarrow 4x = \frac{p^3 + 89^2}{4p} = \frac{p^2}{y} + \frac{89}{p}$$

dyfernhal w. r. ty

=)
$$\frac{4}{p} = \frac{dP}{dy} \left(\frac{2P}{y} - \frac{8y}{p^2} \right) + \frac{8}{p} - \frac{P^2}{y_2}$$

=)
$$p dp = \frac{dy}{2y}$$

=) $log p = \frac{1}{2} logy + log c$
=) $P = c \sqrt{P_1}$ Put in ensinal PAG
=) $P = c \sqrt{P_1}$ Put in

$$=) \quad \frac{1}{2} \left[4x + 8y^2 = 0 \right]$$

Solve by Laplace

$$(D^4 - 1)y = 1$$
, if $y = Dy = D^2y = D^3y = 0$ when $t = 0$.
 $(D^4 - 1)y = 1$

applieg laplace transformation L(D4y)-L(g) = L(1)

$$\Rightarrow Y = \frac{1}{P(P^4-1)} = \frac{1}{P(P^2-1)(P^2+1)}$$

$$= \frac{e^{t} - e^{t} e^{x} - e^{-t} \cdot e^{x}}{2} \left| e^{t} - \frac{e^{t}}{2} e^{t} \left(\frac{e^{t} \cos x}{2} \right) + \sin x \right)$$

$$+ \frac{e^{-t}}{2} e^{2} \left(\cos x + \sin x \right)$$

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$$y(t) = \left[\frac{e^{t-\lambda} - e^{-t+\lambda}}{2} - \frac{e^t}{\lambda} e^{-\lambda} \left(\frac{\sin \lambda - \cos \lambda}{2} \right) + \frac{e^{-t}}{\lambda} e^{-\lambda} \left(\frac{\cos \lambda + \sin \lambda}{2} \right) \right]^t$$

$$= -\frac{1+e^{t}-e^{1}+e^{t}}{2}-\underbrace{e^{t}}_{2}\underbrace{\underbrace{e^{t}(sint-cos)t}_{2}}_{2}+\underbrace{e^{t}}_{2}\underbrace{(-1)}_{2}$$

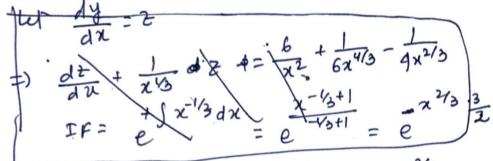
$$= -\frac{2}{2} + \frac{e^{t} + e^{-t}}{2} + \frac{1}{4} \left(2\omega st \right) + \frac{e^{t} + e^{-t}}{(-4)}$$

$$= -1 + 2\omega t + \frac{\omega st}{2} + \frac{1}{2} (\omega sh t)$$

$$y(t) = \frac{1}{2} \cosh t + \frac{\omega dt}{2} - 1$$

Solve by Normal Method the following differential 5) equations:

$$\frac{d^2y}{dx^2} + \frac{1}{x^{1/3}}\frac{dy}{dx} + \left(\frac{1}{4x^{2/3}} - \frac{1}{6x^{4/3}} - \frac{6}{x^2}\right) = 0.$$



$$P = x^{-1/3}$$

$$U = \int_{e}^{-1/2} P dx = e^{-\int_{e}^{-1/3} dx} = e^{-\frac{1}{2} x^{2/3}}$$

$$= e^{-\frac{1}{2} x^{3/3}}$$

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$$\Gamma = -\frac{6}{2}$$

$$S = \frac{1}{4} = 0$$

$$S = \frac{L}{U} = 0$$

$$\frac{d^{2}V}{dx^{2}} + \frac{L}{V} = 0$$

$$\Rightarrow \frac{d^{2}V}{dx^{2}} - \frac{6}{x^{2}}V = 0$$

$$\Rightarrow \frac{d^{2}V}{dx^{2}} - \frac{$$

$$V + 6 \log x = c_1 x + c_2 (b^2 - b^2 - 6) V = 0$$

$$\Rightarrow (b^2 - 3b^2 + 2b^2 - 6) V = 0$$

$$\Rightarrow (b^2 - 3b^2 + 2b^2 - 6) V = 0$$

$$\Rightarrow (b^2 - 3b^2 + 2b^2 - 6) V = 0$$

$$\Rightarrow V = c_1 e^{3z} + (2e^{3z} + c_2) V = 0$$

$$V = c_1 x^3 + \frac{c_2}{x^2}$$

$$V = c_1 x^3 + \frac{c_2}{x^2}$$



6) Solve the following differential equations:

$$(x\sin x + \cos x)y'' - x\cos x \cdot y' + y\cos x = 0.$$

$$y'' - \frac{x \omega x}{x \sin x + \cos x} \quad y' + \frac{y \cos x}{x \sin x + \cos x} = 0$$

$$P = -\frac{x \cos x}{x \sin x + \cos x} \quad Q = \frac{1}{x \sin x + \cos x}$$

$$P + Qx = 0$$

:. u= x is one solution where y=ux

$$\Rightarrow \frac{dw}{dx^2} + \left(\frac{-x\cos x}{x\sin x + \cos x} + \frac{2}{x}\right) \frac{dw}{dx} = 0$$

$$\Rightarrow \frac{dW}{dx} + \left(\frac{-x\cos x}{x\sin x + \cos x} + \frac{2}{x}\right) \frac{dW}{dx} = 0$$
Let $\frac{dV}{dx} = E$

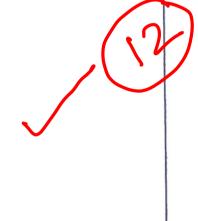
$$\Rightarrow \frac{dz}{dx} + \left(\frac{2}{x} + \frac{-x\cos x}{x\sin x + \cos x}\right) dz = 0$$

$$= \frac{d^2}{2} + \left(\frac{1}{2} - \frac{x \cos x}{x \sin x + \cos x}\right) dx = 0$$

$$\Rightarrow z = \frac{c(x \sin x + \cos x)}{c(x \sin x + \cos x)}$$

$$\Rightarrow \frac{dv}{dx} = c \frac{dv}{dx} - \frac{\cos x}{x}$$

$$\frac{1}{\sqrt{1 + \frac{1}{2}}} = \frac{1}{\sqrt{1 + \frac{1}{2}}}$$



Use the variation of paraneters method to show that the 7) solution of equation $dy dx^2 + k^2 y = \phi(x)$ satisfying the initial conditions y(0) = 0, y'(0) = 0 is y(x) = $\frac{1}{\nu} \int_0^x \phi(t) \sin k(x-t) dt.$

$$\frac{d^{4}y}{dx^{2}} + k^{2}y = \phi(x)$$

Homogenous Part of D.E.

=> m2+62=0=> m=±ik.

Note let Up = ACOSEXT BSINEX

$$A = \int \frac{\sin(x) \cdot \phi(x)}{x} dx = \int \frac{\sin(x) \cdot \phi(x)}{x} dx$$

: 4p = coskx (-sinkx o(n) dx + oossinkx (coskx ocn) dx.

DCF = 9 COSKX +CZ SBAKX

$$3cF = 4 \cos kx + cz \sin kx$$

$$(x-t)dt$$

$$(x-t)dt$$

$$(x-t)dt$$

$$= \frac{1}{k} \int_{0}^{x} \phi(t) \sin(k(t-t)) dt$$

$$= \frac{1}{k} \int_{0}^{x} \phi(t) \sin(k(t-t)) dt$$



8) Solve
$$(D^2 + a^2)y = \tan ax$$

$$= \frac{1}{(D+ia)(0-ia)} tanax$$

$$y_{p} = \frac{1}{2^{\circ}a} \left(\frac{1}{D - ia} + anax - \frac{1}{D + ia} + anax \right)$$

y= c, weart czsinax + 1 iaz sinax by secantanax

9) Solve
$$(D^3 + 1)y = e^{2x}\sin x + e^{x/2}\sin(\sqrt{3}x/2)$$
.

$$(D^{3}+1)y = e^{2x} \sin x + e^{x/2} \sin(\frac{\sqrt{3}x}{2})$$

$$m=-1$$
, $\frac{1+\sqrt{3}i}{3}$, $\frac{1-\sqrt{3}i}{3}$

$$4p = \frac{1}{D^3+1} e^{2x} \sin x + \frac{1}{D^3+1} e^{x/2} \sin \frac{\sqrt{3}x}{2}$$

$$| \sum_{\substack{D^{3+1} \\ = e^{2x} \cdot 1 \\ D+2^{3+1}}} e^{2x} \sin x = e^{2x} \frac{1}{D^{5+8+3D^{2}\cdot 2+3D\cdot 9+1}} \sin x$$

$$= e^{2x} - \frac{1}{3 + 8 - 6 + 3|20+|} \sin x$$

$$= e^{2x} \frac{1}{11D + 3}$$

$$= e^{2x} \frac{11D - 3}{\# |21D^2 - 9|}$$

$$= e^{2x} \frac{(11 \cos x - 35i n 4)}{-130}$$

$$= e^{2x} \frac{110-3}{41210^2-9} \sin x$$

$$= e^{\frac{2}{1}} \frac{1}{\left(D + \frac{1}{2}\right)^{3} + 1} \frac{1}{2} \frac{1}{2} = e^{\frac{2}{1}} \frac{1}{D^{3} + \frac{1}{4} + \frac{3}{4}D + \frac{3}{4}D^{2} + 1} \frac{1}{2} \frac{1}{D^{3} + \frac{1}{4} + \frac{3}{4}D + \frac{3}{4}D^{2} + 1} \frac{1}{2} \frac{1}{D^{3} + \frac{1}{4}D + \frac{3}{4}D^{2} + 1} \frac{1}{2} \frac{1}{D^{3} + \frac{1}{4}D^{3} + \frac{1}{4}D^{3} + \frac{1}{4}D^{2} + \frac{1}{4}D^{3} + \frac{1}{4}$$

$$= e^{\frac{1}{2}} - \frac{1}{\sqrt{2}} + \frac{3}{4} + \frac{1}{8} - \frac{3}{2} \times \frac{3}{4} + 1$$

$$= e^{\frac{1}{2}} - \frac{1}{\sqrt{2}} + \frac{3}{4} + \frac{1}{8} - \frac{3}{2} \times \frac{3}{4} + 1$$

$$y = c_1 e^{-x} + e^{\sqrt{2}x} (c_3 \sin \frac{\sqrt{3}}{2}x + c_2 \cos \frac{\sqrt{3}}{2}x) - \frac{e^{x/2}}{3}x \sin \frac{\sqrt{3}}{2}x + \frac{x}{3})$$

$$+ \frac{e^{2x}}{130} (5\sin x - 11\cos x)$$

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10) Solve
$$(x^{2} + y^{2})(1 + p)^{2} - 2(x + y)(1 + p)(x + yp) + (x + yp)^{2} = 0$$

Put $x + y = u \Rightarrow d + dy = du$
 $x^{2} + y^{2} = 2y$
 $\Rightarrow 2x + 2dy = 2dy$
 $\Rightarrow 1 + p = du$
 $\Rightarrow x + py = dy$
 $\Rightarrow 1 + p = du$
 $\Rightarrow x + py = du$
 $\Rightarrow 1 + p = du$
 $\Rightarrow x + py = du$

$$= \frac{x^{2} + y^{2} + \sqrt{(1+h^{2})}}{(x+yp)^{2}} + \frac{2(x+y)(1+p)}{(x+yp)} + 1 = 0$$

$$=) u = PV + \frac{1}{2P}$$

=)
$$PV \cdot 2' = 2u P + 1 = 0$$

=) $u = PV + \frac{1}{2P}$
Clarent form
:. $u = CV + \frac{1}{2C}$
=) $(x+y) = 2c(x^2+y^2) + \frac{1}{2C}$